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Command and Control Systems Requirements Analysis

Volume 2
Measuring C² Effectiveness with
Decision Probability

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ADMINISTRATIVE INFORMATION

The work reported herein was conducted for the Deputy Chief of Naval Operations (Naval Warfare) during FY 90 as part of the Naval Warfare Analysis Program. This document presents the results of a collaborative effort involving SAIC and NOSC Code 171 Systems Analysis Group, personnel.

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COMMAND AND CONTROL
SYSTEMS REQUIREMENTS ANALYSIS

VOLUME 2
MEASURING C2 EFFECTIVENESS
WITH DECISION PROBABILITY

SEPTEMBER 1990

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SECTION 1.0 INTRODUCTION

Military and Naval Operations are a complex set of activities involving organizations of people and equipment in conflict with another such organization. Analysis of the overall effectiveness of the opposing forces is a difficult task. Part of the problem is the enormous complexity of the interaction. Another difficulty is the lack of means to evaluate the contribution of Command and Control (C2)* to the eventual outcome.

Descriptions of large scale complex systems are very difficult to derive because of the many levels and frames of reference required for understanding. These many levels are necessary because of the nature of the problem and because we are limited in our ability to comprehend more than a few aspects of a problem at the same time. The concepts being presented here will not make the problem simple, but some simple tools will be provided, which, when applied consistently, can be used to gain insight into the problem and, when applied repetitively, iteratively, or recursively, can be used to describe a complex system in more manageable terms. These insights are based on common sense and well-known ideas, but this presentation provides a framework of conventions to clarify relationships and identify similarities and differences among a few fundamental concepts about the nature of systems. The role of decision making in systems is also stated in a way that can be applied to all aspects of the problem.

1.1 BACKGROUND

Naval Warfare needs are described in a series of Top Level Warfare Requirements (TLWRs) documents. TLWRs have been developed by the Office of the Chief of Naval Operations (OPNAV) for some Warfare Mission Areas (WMA), as well as for Electronic Warfare (EW) and for the Carrier Battle Force (CVBF). TLWRs are now being addressed at an even higher level, that of the Functions of the Navy, beginning with Sea Control and, subsequently, for Power Projection.

The TLWRs for Sea Control are expressed in terms of Mission Success Criteria (MSCs). These are statements of objectives to be achieved in various mission situations. The ability to achieve the MSCs is expressed as a combination of Required Capabilities (RCs) in the various WMAs. The RCs are, in effect, sub-objectives that would lead to the accomplishment of the MSCs. In the TLWRs for WMAs, these RCs become MSCs and, to support them, there is a set of RCs for platform mobility and sensor and weapon systems. C3I requirements have been stated subjectively in qualitative and quantitative terms in the TLWRs and other references, but not in a way that exhibits the contribution of C3I to Warfare goals.

* For the purposes of this report, the acronyms C2, C3, and C3I will generally refer to the processes of Command and Control (C2); Command, Control, and Communications (C3); and Command, Control, Communications, and Intelligence (C3I); while the word "systems" will be appended if necessary to distinguish physical resources from the processes. C2 involves decision making and the total information processing that supports it. C3 adds the information exchange process among decision-making elements. C3I represents an emphasis on processing and exchange of Intelligence data within the C3 process, not on the collection of Intelligence data. Similarly, the Surveillance data collection is not included within every C2 process, unless it is the Mission of that element.

Within the organization of the Deputy Chief of Naval Operations for Naval Warfare, the Director, Electronic Warfare (EW), C3I, and Space Warfare (OP-76) is responsible for the C3I Warfare Support Area Appraisal, a major component of the Navy's Planning, Programming, and Budgeting System. OP-76 is also responsible for the administration of Team "C", which is guiding the development of the Navy's Battle Management C3I Master Plan. OP-76 is evolving a methodology for analyzing C3I Warfare requirements in support of these efforts. Previous work has resulted in a C3I Operational Requirements Framework (reference (a)), hereafter referred to as the Framework, and the conduct of Workshops on Tactical C3I Requirements and Deficiencies for Power Projection and Sea Control. This task is intended to extend and enhance the Framework in support of the next cycle of assessment and master plan development.

Within the Space and Naval Warfare Systems Command, the Warfare Systems Architecture and Engineering (WSA&E) Directorate (SPAWAR-30) directs the development of architectural descriptions and assessments of current and future Naval Warfare Systems. The process is governed by the issuance of the TLWRs by OP-07. In response, the Architecture team is attempting to devise a means of providing a traceable accounting of the relationship between system performance and the TLWR. This has given rise to the development of a methodology for Architectural description, modeling, and assessment that is ongoing. This methodology addresses Operational Functions, System Capabilities, and Force Performance Measures. The Warfare Mission Support Areas Division (SPAWAR-312) has solicited the Naval Ocean Systems Center (NAVOCEANSYSCEN) to lead a team of Navy Laboratories to address C3I Architecture issues. This report provides support and guidance in coordination with that effort.

The objective of this effort is to develop a hierarchical multi-level analysis structure of functions and metrics, down to the Force level, that relates Operational Functions and Resource Capabilities to Mission Success Criteria, Required Capabilities, and Force Performance Measures, and describes how these depend on Mission context. The analysis structure will make evident the contribution of C3I, embedded in the operation, to effect Mission Success. The first volume, subtitled The Hierarchy of Objectives Approach (reference (c)), addresses an approach to functional analysis of Naval Warfare at the top levels, addressing military objectives and mission area characteristics to the intra-task force level, with a focus on how C2 effects results. This volume (vol. 2) presents methods for mathematically relating capabilities and objectives at those levels. This metric analysis will be based on a common measure (conditional probability) to quantify the effect of dependency among functions at all levels of the hierarchy. Volume 3, subtitled Command and Control System Functions in the Hierarchy of Objectives (reference (d)), focuses on functional and metric considerations at the system level.

1.2 OVERVIEW OF APPROACH

1.2.1 Role of Hierarchy of Objectives

A Hierarchy of Objectives can be stated in terms of Missions, Functions, and Tasks. For a particular Force or System, its Functions are the activities it performs in order to accomplish its Mission. Its Tasks are its subfunctions, which are performed by its parts or subsystems. Mission objectives are based on achieving a preferred set of outcomes, which are particular states of the enemy's forces and ones' own, as well as the state of the environment, e.g., occupied territory. These objectives may support a higher objective, such as the capitulation of the enemy. The strategies, operations, tactics, and procedures used by each Force are a hierarchy of functions or processes that correspond with their hierarchy of objectives. The sub-objectives are to achieve favorable outcomes of the functions, i.e., those outcomes that contribute to achievement of the outcomes stated in the Mission objectives. Functions/objectives at each level may support several of those at a higher level or of a larger Force. The role of the Hierarchy of Objectives is to define the set of functions and their favorable and adverse outcomes, not only at the highest level, where

the TLWRs establish Mission Success Criteria, but also for Required Capabilities and below. The sets of outcomes form the basis for defining a measure of potential achievement of the objectives.

1.2.2 Role of Decision Making in the Hierarchy of Objectives

Decision making is a function that is performed at all levels of the Hierarchy of Objectives. It is the function that determines which objectives to pursue, which functions to perform and which resources to use and when. The purpose of decision making is, therefore, to allocate resources to perform functions in support of higher objectives. The decision process consists of the performance of decision functions that involve interpreting information or choosing courses of action. These functions, called Command Functions, have decisions as their outcomes. They are described in detail in Appendix A of reference (c), which provides an updated version of the Command Process Model (reference (a)) previously used in OP-76 assessments. Reference (b) is a stand-alone version of Appendix A of reference (c).

1.2.3 Elements of System Description

A method of describing systems of all magnitudes was defined in reference (c) and is summarized in section 2.0. The approach is similar to techniques in object-oriented programming. Objects are described by their states or attributes and by functions that are relationships between the states. Going beyond the ideas of object-oriented programming, this method recognizes that functions represent a causal conditioning between the states, i.e., that the value of a state (outcome of the function) depends on the values of other states. This conditioning can be the basis of a graphic technique for depicting architectural structure. The Hierarchy of Objectives is such a structure of functions to be performed in order to achieve preferred values of states. The conditioning relationship also provides a foundation for assigning a measure of likelihood to the values of a state that is dependent on the values of the other states. This can be a deterministic or stochastic likelihood that can be expressed as conditional probability or related measures.

1.2.4 Probability as a Common Measure

Probability is a term that may mean randomness or proportionality or tendency. Mathematically, it is simply a measure whose value is in the range, 0 to 1. Any of the other meanings can apply, subjectively, depending on the context of the problem. The simple idea of defining the "probability of achieving outcomes" as a common measure is a very natural one. It is also very useful and necessary to have this standard criterion at each level of description of the objectives and processes under consideration. This approach makes probability a common denominator to relate other measures of goodness (attributes and "-ilities"), along with the other universal variable, time. Probability is a non-dimensional measure, but it is referenced to the dimensions of the attributes or states that characterize outcomes.

Due to recent results in Conditional Probability and system analysis techniques, this approach will provide the means to understand the relationship between system performance and Mission Success Criteria. The probability measure is not an isolated quantity. It is a surrogate for related criteria and capabilities. The measure is a conditional probability, where the dependencies of various outcomes of functions on those of other functions is reflected in the conditioning of the probability measure. The capabilities are statements of desired outcomes conditioned on the context of the requirement statement. After translating those capabilities into probability space, the Hierarchy of Objectives provides a roadmap for mathematically combining the probabilities of the lower levels to produce a probability of the outcomes at the highest level, where the ultimate measure is the "probability of achieving preferred outcomes".

1.2.5 Decision Probability as a Measure of C2 Effects

Probability is also used to relate the effects of C2 to the accomplishment of the Mission. Since decisions are the outcomes of the Command Process, the measure of the contribution of decision making is the probability of making particular decisions. The functions actuated by decisions are conditioned, in part, on the making of those decisions and the decisions are conditioned on the outcomes of the decision subfunctions and the information available. The effect of making a particular decision or another, or of its timeliness, is reflected in the resulting probability of achieving the top level objective, all other conditions and probabilities being kept the same.

1.3 ORGANIZATION OF REPORT

Section 2.0 provides a summary of the system description concepts developed in reference (c). Section 3.0 defines the role of probability in connecting together the chain of events that constitute the interactivity of systems and their outcomes. Section 4.0 discusses how the common measure of probability can be applied to the Hierarchy of Objectives, starting with MSCs and RCs. These criteria and capabilities may need to be translated into probabilities as demonstrated in section 5.0. The relationships of the ideas developed in this methodology to other system analysis tools is provided in section 6.0 and conclusions are presented in section 7.0. Appendix A provides additional mathematical material in support of section 3.0. Appendix B is a numerical example of using conditional probabilities to examine combat outcomes influenced by decision probabilities.

SECTION 2.0 SUMMARY OF SYSTEM DESCRIPTION CONCEPT

The Hierarchy of Objectives Approach (reference (c)), provides a way of describing systems with a few commonly defined concepts. First, that there are elements (objects) that are described by their states (attributes), which may take on a range of values. The values that a particular state may realize must be mutually exclusive and exhaustive, meaning that the state may not have two values at the same time and that all possible values have been listed. Second, that the behavior of elements is embodied in processes (functions) that result in outcomes, which are realizations of a combination of values of the states. Each process is a relationship between its outcomes and the events (outcomes of other processes) that cause them. The collection of attributes and processes represents the nature of the object. This concept is similar to that of object-oriented programming.

Time is associated with each attribute, explicitly or implicitly. The value of the state may depend on the value at a previous time through a conditioning relationship. Processes then reflect a dependence on the time difference.

Decision making is a special process that results in outcomes called decisions, which are realizations of information states. The decisions may be inferential (deciding to believe) or intentional (deciding to do). Organizational elements are special elements that perform decision making functions. Actions are functions that implement "intentional" decisions. The outcome of the action (turn on, issue directive) is a control of another process that may be influenced by other controls or other effects.

Resources are described by physical states and outcomes, which are conditioned on other physical events and the control events (actions) caused by decisions (information states). Some outcomes (emissions) propagate through the environment (as states of the environment) to become stimuli to various elements. Stimuli may also impact other physical states. Sensing is the function that translates stimuli and other physical states into information states.

An object is fully defined when all its attributes (or states) and processes are defined. This is synonymous with defining all the possible outcomes of its functions and, therefore, it is defined by its functions. These are inherently concurrent definitions.

A function or process is fully defined when all its outcomes are defined and all its antecedent events are defined. If there are intermediate outcomes, these identify the boundaries of subfunctions. Thus, event sets (states) decompose in a complementary way with their corresponding functions.

For purposes of a particular analysis, the previous statements may be modified to read: "A function or process is sufficiently defined when all its relevant outcomes and antecedent events are defined," and similarly for objects. Primarily, relevance is established for the outcomes related to the objectives of the analysis. Relevance is also determined by whether the events influence other relevant events or are necessary for connecting functions together.

A picture of a system can be developed where the states are represented by black dots (or nodes of a graph). Each dot also represents the set of outcomes for that state. The states are connected by arrows that indicate a causal dependence of the node at the head of the arrow on the nodes at the tail. The arrows make this a directed graph. By connecting together those events that influence other events or outcomes, the resulting graph represents a functional architecture of the system. Since the influence in this case is causality, the directed graph is called a causality net. This graph will also serve as a depiction of the mathematical relationship among the functions and their events.

SECTION 3.0 OVERVIEW OF CONDITIONAL PROBABILITY

The approach to developing Decision Probability Measures depends heavily on the concept of the Algebra of Conditional Objects devised by Goodman and Nguyen (reference (e)) and the Conditional Probability Logic that can be derived from it (references (f) and (g)) and implemented in a prototype model (reference (h)). The mathematical concepts from reference (h) are summarized in Appendix A, and their implications for system analysis are discussed below.

3.1 CONDITIONAL ALGEBRA

The Conditional Algebra is defined on a set of mathematical forms or abstractions symbolized by $(X | Y)$ or "X given Y". The variables, X and Y, can take on continuous values, x and y, or discrete values, x_i and y_j , in some range. The consequent, X, is said to be conditioned on the antecedent, Y. If this relationship is causal, Y causes X. The converse relationship involves inferencing, i.e., $(Y | X)$ means Y is inferred from X. Although these two relationships can be mathematically connected by Bayes' Rule, for the purposes of system analysis, only causal relations and directions will be used. The Conditional Algebra is a generalization of ordinary set algebra. The ordinary set, (X), can be represented by the conditional set, $(X | W)$, where W represents all sets (or variables), including X, in the space under consideration, i.e., the whole space.

The variables, X and Y, can also be compound variables. That is, they can represent all combinations of two or more separate variables, symbolized by $(X | Y) = (A, B | C, D)$ or "A and B' given 'C and D'". For example, if X is position and Y is velocity, they are made up of two variables each: LAT (latitude) and LON (longitude) for X, and LAT RATE (latitude rate) and LON RATE (longitude rate) for Y. Then X and Y may be considered vector variables. The variables, LAT and LON, can take on values, lat and lon, in the ranges (-90 to 90) and (-180 to 180), respectively. The variable, X, can take on values of $x = (lat, lon)$.

Conditional independence is a way of expressing the lack of conditioning of a consequent variable on other variables. The expression, $(X | Y, Z) = (X | Y)$, means X depends on Y but not on Z; for example, $(LAT | LAT RATE, LON RATE) = (LAT | LAT RATE)$.

When the values in the ranges of all variables have been stated in a mutually exclusive and exhaustive manner, conditional forms can be combined in a simple way. The two forms, $(A | B, C, D)$ and $(B | C, E)$ can be combined (joined or intersected) to form $(A, B | C, D, E)$. The two consequents become a compound consequent and the remaining antecedents become a compound antecedent, while B is removed from the antecedent list. Although both the original forms depended on C, while one was independent of D and the other of E, the result depends on all three. Note that since B was an antecedent of A in the first form, A cannot be an antecedent of B in the second, since then there would be circular causality. The conditional algebra also allows the combination of two forms where neither consequent appears in the antecedent of the other. For example, $(A | B)$ and $(C | D)$ can be joined to form $(A, C | B, D)$ directly, since A and C are mutually conditionally independent.

Part of a compound consequent can be removed from the analysis by taking the union of the conditional form over the values of the variables to be removed. This operation is the basis for integration of the numerical measure of probability over the range of the variables, described below.

Time, when relevant, is associated with every variable in the list of consequents and antecedents, although it may not be written explicitly. This time need not be the same time for each variable. A

time tag associated with a variable, such as a state, distinguishes it from the same variable at a different time, while the name of the variable and its range of values are otherwise identical. Thus $X(t)$ and $X(t+s)$ are different variables but relate to the same attribute, X , with the same range of values.

3.2 RELATIONSHIP OF ALGEBRA TO SYSTEM DESCRIPTION ELEMENTS

The conditional form provides an algebraic (qualitative, syntactic) method for representing the relationship of objects, attributes, functions and outcomes. In describing objects, each attribute (state, stimulus or emission) is a variable (or dimension) of the analysis. Each function or process is a causal conditioning relationship. The role of conditioning is to establish the context for the describing the outcomes. The consequent variables of a function represent its outcomes, while its antecedents are the outcomes of other functions. For example, position is an outcome of a movement function conditioned on velocity outcomes of a propulsion/steering function, which are conditioned, in turn, on helm and throttle controls, as well as the state of the propulsion/steering systems: on/off, none/one/two engines operating, etc.

At the force level, the dimensions of the analysis may consist of the number of each type of platform, their formation, readiness, position, velocity, posture, etc. Each of these is dependent on some of the others, but each of them is also dependent on some decision, such as how many platforms to assign to the Force. They may also be dependent on factors in the environment, on actions of the enemy or contextual situation, such as level of conflict. The number of combinations of all variables is unimaginable. Fortunately, the outcome variables (consequents) of each function only depend on a "few" antecedents in a direct way. The position variable is only indirectly affected by the throttle variable through the intermediate variable of velocity. The dependence (causality conditioning) relationship is a principal part of the knowledge base needed to analyze a system. This dependency knowledge is evident in the equational relationship between the variables, that is, the equation that describes the behavior of the outputs (consequents) of a function based on its inputs (antecedents) and its parameters, e.g. $x = f(y,a)$, where a is a parameter. The equation makes it obvious that x does not depend on z , directly, but only indirectly, if y depends on z . This knowledge reduces the overall complexity of the analysis.

Time plays an important role in the relationship of the variables. For instance, "current" position also depends on "previous" position and on the history of velocity between then and now. In general, a consequent variable often depends on its previous value, the time between outcomes and the history of its antecedents during the interval. The number of forces remaining on both sides in a conflict depend on the number of forces at the beginning and the activity in between.

3.3 CONDITIONAL LOGIC AND PROBABILITY

Probability is a measure that can be applied to the members of a set (values of a variable), such that the value of the measure is between 0 and 1 for each value of the variable, and the sum of the values of the measure over all the values of the variable equals 1. Conditional probability is a generalization of ordinary probability. Conditional probability assigns a measure between 0 and 1 to each value of the consequent variable (or combination of values for compound variables) for each value (or combination) of the antecedent variable(s). The summation of the measure over the consequent variable(s) must be equal to 1 for each combination of the antecedent(s). The focus of the measure on the antecedent provides a means to express the effects of context and causes on the probability of outcomes. A conditional probability is symbolized by $\Pr(X | Y)$ to represent the whole distribution. The individual probability values can be written, $\Pr(x | y)$. The use of this measure and the operations of multiplication and addition (or integration), when applied to the set of conditional forms, defines a conditional probability logic.

3.3.1 Deriving Conditional Probability Distributions and Models

If the equational dependence of a variable on other variables is known (see section 3.2), not only is the conditioning relationship known, but the probability function can be derived from it. Any equation may involve deterministic terms or they may have additive or multiplicative "noise" terms, or a combination. There are ways, by resorting to intermediate dummy variables, to convert the equation into one or more equations, where the "noise" is additive or nonexistent. An individual function will be in the form, $x = f(y,a) + n$, where n is the noise term. This can be rewritten as $x - f(y,a) = n$. If the distribution of the noise term, $Pr(N)$, is part of the knowledge base, then the conditional probability, $Pr(X | Y)$ is equal to $Pr(N)$, with $pr(x | y) = pr(n)$ for $n = x - f(y,a)$ for each combination of x and y . This is referred to as the regression relation. If there is no noise, then the relation is deterministic and the probability of a value of $X = x$, given a value of $Y = y$, is 1 if $x = f(y,a)$ and 0 if it is not. Since the equational form of the relationship is a model of the process, it is equivalent to saying that the conditional probability form is a model, too. The conditional form itself is also an equivalent qualitative model. (Note that the form of the function $f(y, a)$ and even the form of the noise distribution, may be different for different values of y . It may also be the case that the parameter, a , is a variable. Then it should be treated as a state.)

Time may be a parameter of the equational relationship. This is related to the time associated with the states on each side of the conditioning form and is reflected in the nature of the probability function.

3.3.2 Aggregating Models

Small models can be combined in a simple way to form larger models. Since the values of the variables are assumed to be mutually exclusive and exhaustive, the product of two conditional probabilities is found by multiplying the probabilities of each combination of both the consequent variables and the antecedents. The expression for the combined conditional probability is the same as that of combining the conditional form in the algebra, e.g., $Pr(A | B,C,D) * Pr(B | C,E) = Pr(A,B | C,D,E)$. This is found by multiplying $pr(a | b,c,d) * pr(b | c,e) = pr(a,b | c,d,e)$ for each combination of a,b,c,d and e . The position and velocity models can be combined, for example, by $Pr(LAT | LAT RATE) * Pr(LON | LON RATE) = Pr(LAT,LON | LAT RATE, LON RATE)$. Note that if there is noise in the two relations and it is correlated, they could not have been separated into two conditional forms in the first place, but coupled into the latter form from the beginning. If the noise were uncorrelated, the smaller forms would be admissible.

Larger models can be simplified to more compact models by removing unwanted variables through integration or summation over their entire range. The model, $Pr(A,B,C | D)$, can be reduced to $Pr(A | D)$ by integrating over B and C . This marginal conditional model is the simplest way to obtain an aggregate model. One may also define a variable, X , that has sets of combinations of other variables, Y and Z , as elements. The deterministic model, $Pr(X | Y,Z, \bullet)$, can be used to multiply models of $Pr(Y | \bullet)$ and $Pr(Z | \bullet)$, where (\bullet) represents any other conditions (variables). Then $Pr(X,Y,Z | \bullet) = Pr(X | Y,Z, \bullet) * Pr(Y | \bullet) * Pr(Z | \bullet)$ and, by integrating over the ranges of Y and Z , the aggregate model, $Pr(X | \bullet)$, is obtained. For example, if X can be SUCCESS or FAILURE, $Pr(SUCCESS | y \leq \text{loss limit for } Y, z \leq \text{loss limit for } Z, \bullet) = 1$, but $Pr(SUCCESS | y > \text{loss limit for } Y \text{ OR } z > \text{loss limit for } Z, \bullet) = 0$, then $Pr(SUCCESS | \bullet)$ is the sum of the probabilities that y and z are both less than or equal to their respective loss limits for the scenario defined by the conditions, (\bullet) .

3.3.3 Completeness and Consistency

By keeping track of the full range of the variables in a mutually exclusive and exhaustive way, the analysis maintains a complete and consistent logical structure. It is complete in the sense that it involves all combinations of conditions as described in the analysis and it is consistent in that the

sum of probabilities adding to 1.0 is a test to check that there is no contradiction in the assertions of probabilities that would add to more than 1.0 or less than 1.0.

For example, consider the idea of detection. This is an event, but sometimes it is the name of a function. This can be misleading, since a false alarm is another event that occurs in the process. A complete analysis must account for both events. The complete outcome state is "Declare a target present or not present". The process is conditioned on whether or not a target is actually present. This highlights the idea of context dependence. The classic "probability of detection" is the case of declaring a target present, given there is a target present, and the "probability of false alarm" relates to declaring a target present, when this is not the case. The complete probability of declaring a target present is:

$$\begin{aligned} \text{pr(Declare present)} &= \text{pr(Declare present} \mid \text{Target present)} * \text{pr(Target present)} \\ &+ \text{pr(Declare present} \mid \text{Target absent)} * \text{pr(Target absent)} \end{aligned}$$

This is complete because both cases of the antecedent, Target present and absent, were considered. To ensure consistency, both cases of the consequent, Declare present or absent should be calculated. In this case,

$$\begin{aligned} \text{pr(Declare absent)} &= \text{pr(Declare absent} \mid \text{Target present)} * \text{pr(Target present)} \\ &+ \text{pr(Declare absent} \mid \text{Target absent)} * \text{pr(Target absent)} \\ &= \{ 1 - \text{pr(Declare present} \mid \text{Target present)} \} * \text{pr(Target present)} \\ &+ \{ 1 - \text{pr(Declare present} \mid \text{Target absent)} \} * \text{pr(Target absent)} \\ &= 1 - \text{pr(Declare present)} \end{aligned}$$

If the last equality does not hold, the pair of equations is inconsistent. It may be because the conditionals are inconsistent, i.e.,

$$\begin{aligned} \text{pr(Declare absent} \mid \text{Target present)} &\neq \{ 1 - \text{pr(Declare present} \mid \text{Target present)} \}, \text{ or} \\ \text{pr(Declare absent} \mid \text{Target absent)} &\neq \{ 1 - \text{pr(Declare present} \mid \text{Target absent)} \}, \end{aligned}$$

or because the antecedents are inconsistent, i.e.,

$$\text{pr(Target absent)} \neq \{ 1 - \text{pr(Target present)} \}.$$

When outcomes or antecedents have large ranges of variables, this is tested by adding up the probability of the outcomes over all values. This can be done to the distribution of the antecedent, to the distribution of the consequent and to the conditional distribution of the consequent for each value of the antecedent. This ensures that the combination of distributions is consistent. This test is made possible by the requirement that the set of values of a state be mutually exclusive and exhaustive.

3.4 METHOD OF ANALYSIS

3.4.1 Markov Model

The basic approach is based on a Generalized Markov process. This method assumes that (the probability of) the state of a system at some time depends on (the probability of) its state at some previous time and (the conditional probability of) the transition process between them. The distribution of the earlier state is called the prior and that of the later state, the posterior, while the transition model or relation is represented by a conditional distribution conditioning the later state on the earlier. These distributions may be $\text{Pr}(Z(t-s))$, $\text{Pr}(X(t))$ and $\text{Pr}(X(t) \mid Z(t-s))$, respectively. The objective is to find out the posterior distribution, assuming the prior and the model are known.

Then the posterior can be calculated by combining the prior and the model to form, $\Pr(X(t) | Z(t-s)) * \Pr(Z(t-s)) = \Pr(X(t), Z(t-s))$. This can be integrated over the range of the prior, $Z(t-s)$, to get $\Pr(X(t))$.

3.4.2 Utilizing Model Decomposition

The model may not be known as a single equation or distribution, but may need to be built up from smaller models. It may be that the models, $\Pr(X(t) | Y(t-r))$ and $\Pr(Y(t-r) | Z(t-s))$, are known. Then the two smaller models can be combined and integrated over the variable, $Y(t-r)$, to yield the overall model, $\Pr(X(t) | Z(t-s))$. The decomposition of the states and processes from the high level model to smaller models is directly related to the hierarchy of objectives. The method of combining models provides the aggregation up from lower level models.

The states, $X(t)$, can be called objective states and can be defined by the top level of a hierarchy of objectives. The states, $Z(t-s)$, are the initial states and represent the assumptions, constraints, and overall context of the problem. The variables in X and Z are often the same attributes, except for the time, but they also may be different states altogether. For example, they may represent the number of Forces on each side, level of conflict, etc. They may differ in that Z may specify total Forces, while X only counts high value units. The variables in Y are intermediate states that relate to lower level objectives and functions in the hierarchy of objectives. Parts of Y may be posterior states of Z that were not included in X , such as lesser valued units. It may be necessary to decompose the states of Y and its processes, represented by the model, $\Pr(Y(t-r) | Z(t-s))$, until models with known properties can be invoked. This assumes that the aggregation relationships are also among the set of smaller models. (These aggregation relationships are models that are high in the hierarchical structure; the term small does not refer to the level of the function.) Some of the smaller models have to have the initial states as their antecedents.

Time is a critical determinant of the nature of the large and small models. The size of s and r , in the above discussion, can totally change the structure of the submodels, their relationships and the probabilities involved. The beginning time of the analysis is established by the initial condition assumptions. The states of the initial conditions are the initial context. Additional context may be specified at later times, as boundary conditions, such as, remaining within some geographical region. Any of these context conditions may be assigned a probability distribution as its prior. But, more often, a single combination of the prior states is specified by setting the probability of that combination to 1.0. This is a single instantiation of the initial conditions. On the other hand, if two models are to be sequentially driven, one by the other, the posterior distribution of the earlier must be the prior distribution of the later.

The aggregation problem can be solved in two ways. In one, the smaller models are combined until the large model is derived and all intermediate states have been removed by integration. The large model is then combined with the prior and the prior is integrated out. The other way is to combine the lower level models with the prior and remove the prior states first, adding and removing intermediate states along the way. Actually, a compromise of these approaches may result in fewer mathematical operations depending on the relative complexity of the models and their relationships.

3.4.3 Complexity Management

The apparent complexity of the problem can be reduced by the recognition of the causal relationships. This allows operations on pairs of smaller models without carrying all the variables of the problem until they are needed. In the meantime, some of the lower level variables can be removed by integration before involving these other variables. For example, to handle the case of:

$$\Pr(W) = \Pr(W | X, Y, Z) * \Pr(X | Y, Z) * \Pr(Y | Z) * \Pr(Z), \text{ summed over } X, Y \text{ and } Z,$$

involves operations with four- and three-dimensional probability models. If each variable were conditioned on only one other, the operation:

$$\begin{aligned}\Pr(W) &= [\Pr(W | X) * \Pr(X | Y) * \Pr(Y | Z) * \Pr(Z)], \text{ summed over } X, Y \text{ and } Z, \\ &= \Pr(W, X, Y, Z), \text{ summed over } X, Y \text{ and } Z,\end{aligned}$$

still involves a four-dimensional probability, but it can be reduced to

$$\Pr(W) = [[[\Pr(W | X) * \Pr(X | Y), \text{ summed over } X] * \Pr(Y | Z), \text{ summed over } Y] * \Pr(Z), \text{ summed over } Z],$$

or the operation:

$$\Pr(W) = [\Pr(W | X) * [\Pr(X | Y) * [\Pr(Y | Z) * \Pr(Z), \text{ summed over } Z], \text{ summed over } Y], \text{ summed over } X],$$

which only involves operations with two-dimensional models. Note that the example was aggregated in the two different ways mentioned in section 3.4.2, first from left-to-right, then right-to-left. The larger the dimensionality of the overall model and simpler the small models, the greater is the reduction in complexity.

3.4.4 Decision Probability

An important category of intermediate states is the decisions that can be made that alter the state of other processes. The decision to put the Force into one or another formation may be critical to the result. The result is conditioned on the decision concerning which formation to use, i.e., $\Pr(\text{result} | \text{decide formation}, \bullet)$. The state of the authority of a decision maker may affect the likelihood that he directs a particular course of action, to the enhancement or detriment of the overall outcome, i.e., $\Pr(\text{decide course of action and resources} | \text{authority, information}, \bullet) * \Pr(\text{result} | \text{course of action, resources}, \bullet)$ yields a dependence of results and decisions on authority and information, as well as on other things. No weapon gets fired unless someone decides to trigger it, and then only if the firing mechanism works. In some situations, it is important to hold fire. Yet there may be an inadvertent discharge of the weapon due to other circumstances. This illustrates the interdependence of objectives, decisions, and physical outcomes. The objective may call for a decision to fire or hold fire, which may or may not result in the preferred outcome.

SECTION 4.0 APPLICATION TO HIERARCHY OF OBJECTIVES

In the hierarchy of objectives, the highest objective is to deter war. This depends on the capability and preparedness of the Armed Forces, as well as political and economic means. The dimension that describes this state is the level of conflict. The objective is achieved as long as the value of that state is Peace. Otherwise, the state is some level of war. The objective of End War is satisfied if the state returns to Peace and the objective of Win War is to terminate it by having the enemy surrender. The latter is a decision state that either side can make. The probability of winning a conflict is, therefore, a Decision Probability, conditioned on the information available about the progress of the war and the prediction by each side of the likelihood of further gains and losses. Both the probability of deterrence and victory depend on decisions. Both decisions depend on the belief of the decision makers of the balance of power at any point in time. If the belief of the instigator is such that winning seems feasible (disregarding the issue of mutually assured destruction), it is up to the respondent to demonstrate superior capability. Both the capability and the perception of it by the enemy are necessary.

4.1 MISSION SUCCESS CRITERIA

The states of the opposing Forces that describe their capabilities include the number of each type of combatant and support units, along with their collective capability. This capability is expressed in Mission Success Criteria (MSCs) in terms of percentages of enemy forces to attrite in a certain time with some confidence and limits of own Forces lost for various levels of conflict. This can be restated as the Probability (confidence) that the enemy's forces are reduced by a percentage and own losses are less than some number, given the number of forces on each side at the start and that someone decided to put the world into a state of war and a decision was made to send these particular forces against each other and how many actually participated in the engagement. In analyzing a particular scenario, it is assumed that the level of conflict is as stated and that all the necessary decisions were made and the number of forces specified. Thus the dependence of the analysis on the probability of all these decisions is hidden by the assumptions of the instantiated scenario. Since the antecedent must state the number of units at the beginning, the percentages can be translated into numbers of units lost by each side. But, in some cases, the number of units available to conduct the operation is a probability of a previous engagement. Thus the number of units to eliminate, to achieve success, may depend on the number that attack. The success of a Battle Force depends on the number of units attacking it and whether the enemy can concentrate their attack or come singly or in pairs. The probability of losing any number of Carriers must be conditioned on the number of attackers. All these possibilities are outcomes of the previous engagement that influence the outcome of the current analysis.

4.2 REQUIRED CAPABILITIES

Required Capabilities (RCs) are also specified in the TLWRs as a way of characterizing the next level of objectives below MSCs. The RCs are descriptions of lower level outcomes that are also conditioned on the scenario assumptions. These are expressed in the same terms as the MSCs but for a smaller context, such as a particular region of the world. They are also expressed in terms of countering the enemy's surveillance, targeting, weapons, and platforms as well as limiting damage when the others are unsuccessful.

The probability of the outcomes of the RCs may be dependent on procedures chosen to employ these capabilities. RCs have not been expressed in terms of the decisions needed to implement the procedures in order to realize the potential of the RCs. These decisions involve a choice of tactics and resource allocation. They must be conditioned on knowledge of the enemy's tactics and capabilities gained from Intelligence and on information about the enemy's disposition from

Surveillance, as well as the decision maker's preference for certain tactics over others, in addition to his authority to carry out those actions. Information about the state of the environment may also be a factor conditioning the decision. The procedures and resource assignments are the consequent dimensions of the decisions, while the other factors are the antecedents. The decision is not conditioned on the number of enemy attacking, but only on the information available about how many there are. The information available is the consequent of the Surveillance, Intelligence and Communications processes. The Surveillance and Intelligence outcomes (reports) are conditioned on the number of enemy involved.

4.3 MULTI-WARFARE EXAMPLE

In reference (c), numerous objective functions were extracted from the TLWRs in the category of Tasks of the Navy, all of which are operational functions that support or are supported by one or more Warfare Mission Areas and Support Mission Areas. Limiting Loss is an objective derived from MSCs supported by AAW, ASUW, ASW, and EW Mission Area capabilities. The achievement of this MSC objective is supported in each of those Mission Areas by their contribution to limiting weapon arrival at targets. This can be accomplished by (1) jamming, decoying, or destroying the weapon (counter-weapon), (2) by jamming, evading, or deceiving the platform carrying the weapon or by destroying the platform (counter-platform) or (3) by countering the platform's, organic or non-organic targeting capability (countertargeting). (Note that there can be an overlap in these approaches, in that countertargeting can be aimed at the weapon's sensors, the platform's, or the non-organic source of targeting information.) The number of weapons reaching an important target is conditioned on the activation of these counter-procedures and the success of these capabilities.

A particularly interesting example of the interdependence of the Mission Areas involves CVBF defense against submarine-launched cruise missiles. This has significant roles for AAW, ASW, EW, and C3I components. AAW is tasked to destroy missiles in flight, ASW and EW(ESM) are responsible for recognizing the presence of the submarine and ASW for destroying it. Acoustic deception and jamming can be used to defeat targeting, as can EW(ECM) methods. The latter also contribute to counter-weapon tactics. C3I provides coordination with informational and decision making support, as well as connectivity. The AAW and EW elements need to know where the submarine is in order to orient their resources. The choice of tactics depends on Intelligence concerning enemy capabilities relative to ours. The decision to employ assets is conditioned on the plan of action and rules of engagement, therein, and demands of other elements of the situation, including concerns for mutual interference or self-attrition.

Appendix B focuses on a part of this interdependent Multi-Warfare example. It addresses the employment of deception to convince the submarine, which is assumed to be attempting to launch, that its target is not a carrier, and whether this countertargeting approach will enhance or diminish the probability of success, on the part of the submarine, of putting a missile in the air. The use of jamming, evasion, or attack on the submarine is not specifically addressed. So the measure to be determined is the probability of a missile being launched, given the enemy is attempting to launch, is not destroyed or evaded, that we know it is there, etc.

Whether or not we decide to use deception devices is an intermediate decision of the problem and the state of our countermeasures is another variable. The belief by the enemy in the classification of a carrier is a state that determines the success of our countertargeting attempt. Other states that are involved include the achievement of launch criteria by the enemy, which may be a determinant of the effectiveness of his counter-countermeasures, when we are employing our deception. The decision to shoot and the ability of the launch system to effect this decision are part of the chain of events necessary for a launch.

When the enemy is unhindered by a deception device, the launch success will not be perfect because there are limits on the enemy's capabilities. When the deception device is on, it may hinder the classification capability. In cases that the enemy is deceived, there will be no launches, since the decision to launch depends on a classification decision of whether it is a carrier. But in cases when the deception fails, the achievement of launch criteria may be diminished by the confusion, enhanced by the reception of the deception signal strength, or remain the same. Since the decision to order a launch depends on the firing solution, so does the likelihood of a missile being launched. Our decision to use countermeasures depends on our knowledge of the likelihood of deceiving the enemy and, failing that, whether it enhances or reduces the quality of the enemy's solution. The interaction of the probability of these decision options is balanced in the analysis of Appendix B, up to the probability of a missile being in the air. It is then the task of AAW to minimize the $\text{Pr}(\text{hit} \mid \text{missile in the air})$, possibly conditioned also on previous warning from ASW or EW sensors.

SECTION 5.0 EXPRESSING REQUIREMENTS WITH PROBABILITY

The use of probability as a common measure, as mentioned in section 1.2.4, provides the means to combine capabilities in a uniform manner using a logic structure that assists in keeping the quantification process complete and consistent. But requirements are usually stated in other terms than probability. In order to put these requirements on a common basis, it may be necessary to translate them into probability expressions related to the attribute mentioned or implied by the requirement statement. For example, the range of a sensor may be stated as a requirement. This may be translated into a requirement about the probability of the detection of a target at that range. Other requirements, such as accuracy, availability, and lethality, can also be translated in terms of probability. The requirement value for the probability is often stated as a confidence value. The time period of the activity is sometimes stated in requirements, but often it is implied. Some antecedent conditions are implied by the context, such as type of target or season of the year, or it may need to be assumed that any or all types or seasons is meant. It is then necessary to use a weighted average over all types or a worst case answer to compare with the requirement probability. The requirement statement should specify which condition is to be assessed and what weights to apply.

5.1 SPECIFYING STATES AND RELATIONS

In order to conduct an analysis, it is necessary to identify all the objects of interest. This entails specifying the relevant states of the top level objectives and the initial states of the assumptions or context. The objects must include those that possess the states of the objective and those of the context. Additional objects and states may be necessary for the complete analysis. In particular, the environment must be an object, if for no other reason than to provide context. This may include political state or other situational considerations. As the objects and states are defined, the causal relationships among them, i.e., functions or processes, must also be defined. This may involve defining additional intermediate states that are the result of having to decompose functions in order to realize functions whose equations are known. Intermediate states may, but need not, constitute lower level goals and objectives. Finally, the conditional probabilities associated with the outcomes of the functions, as conditioned by other states, must be identified. This will be 1.0 or 0.0 for deterministic relationships and a distribution over several values of the consequent for stochastic instances. Different combinations of antecedent values may have either deterministic or stochastic cases in the same functional relationship. For example, speed is zero for sure if damage state is "destroyed". This implies that

$$\text{pr}(\text{speed is } 0 \mid \text{damage state is "destroyed"}) = 1.0,$$

but speed may be random, having some variance around the nominal, if the state of damage is "not destroyed". These speeds, of course, should also be conditioned on the power plant state and the ordered speed.

The decomposition of objectives, states, objects and functions, with associated probabilities, continues until the relevant states of the objective and initial conditions can be tied together with known relationships, including known (or postulated) values of the probabilities. If the intent of the analysis is to assess the contribution of systems, the capabilities of the systems must be part of the structure.

5.2 WARFARE REQUIREMENT EXAMPLES

Warfare requirements can always be stated in terms of a probability of a state, given some context of assumptions and a time period for the accomplishment of the activity or objective, although sometimes this form may seem stilted or stylized. The translation from ordinary language may have to be performed by the analyst. Some examples of such conversions are given below, where the symbol, \Rightarrow , means "translates to".

- a. Force Level objectives (MSCs):
Limit Loss \Rightarrow $\text{Pr}(\# \text{ of units remaining} \geq \text{minimum} \mid \# \text{ of units initially, what scenario?})$
- b. Battle Space:
Range, Volume \Rightarrow $\text{Pr}(\text{Detect or kill or other event} \mid \text{range, volume, what conditions?})$
- c. Survivability \Rightarrow $\text{Pr}(\text{unit is "not destroyed"} \mid \text{activity, what conditions?})$
or $\text{Pr}(\# \text{ survive} \mid \text{activity, what conditions?})$
- d. Availability \Rightarrow $\text{Pr}(\text{unit is "available"} \mid \text{what conditions?})$
- e. Accuracy \Rightarrow $\text{Pr}(\text{error} < \text{max error} \mid \text{quality of information, availability of information})$
- f. Time is included in requirements to specify the span of time to accomplish a goal. Then the requirement is

$\text{Pr}(\text{required outcome occurs by time } (t) \mid \text{what conditions at time } (0)?).$

- g. Another way that the role of time is often expressed in requirements is in terms of "timeliness". This concept is usually based on an implicit requirement regarding another outcome that is dependent on the subject event to which the "timeliness" criterion is applied. Then two probabilities must be combined to result in the overall outcome:

$\text{Timeliness} \Rightarrow [\text{Pr}(X(t) \mid Y(t-r)) * \text{Pr}(Y(t-r) \mid Z(t-s)) \text{ summed over } Y(t-r)]$ is greater than the required $\text{Pr}(X(t) \mid Z(t-s)) = P^\wedge$

where $\text{Pr}(X(t) \mid Y(t-r))$, $\text{Pr}(Y(t-r) \mid Z(t-s))$ and $\text{Pr}(X(t) \mid Z(t-s))$ are as defined in section 3.4.2, P^\wedge is the minimum (goal) probability of the overall outcome. In order for this to be true, the timeliness requirement on the intermediate outcome becomes

$$\text{Pr}(Y(q) \mid Z(q-(s-r))) \geq \text{Pr}(Y(q) \mid Z(q-(s^\wedge-r^\wedge))) \text{ for } (s-r) \leq (s^\wedge-r^\wedge),$$

where $s-r$ is the time to accomplish the intermediate outcome, Y , s^\wedge is the time needed to exceed the goal and r^\wedge is the time remaining to accomplish the next step. That step has to achieve

$$\text{Pr}(X(t) \mid Y(t-r)) \geq P^\wedge / \text{Pr}(Y(q) \mid Z(q-(s-r))).$$

This is a complicated relationship, but it reflects the interdependence of the intermediate and final outcomes and their associated times. The capability of the follow-on process must be known and must be part of the requirement statement in order to define "timeliness".

- h. Although more a system level requirement, "throughput" can be expressed as a probability in two ways:

Throughput \Rightarrow $\text{Pr}(\# \text{ of units processed in time } (t) \mid \text{system operating condition, load demand})$, or
 $\text{Pr}(\text{average time to process one unit} < t / \# \text{ of units} \mid \text{system operating condition, load demand})$.

5.3 C3I REQUIREMENTS

In addressing C3I requirements, the consequents of the causal relationships and conditional probabilities can be related to the outcomes of the subfunctions in the Command Process Model (reference (b)). Since the principal purpose of decision making is to select a course of action (COA) and assign resources to carry it out, the probability of this event is a key one in any model of the process. The functions that support this event produce the current tactical picture and an assessment of the situation it represents. Obtaining data, including direction from higher authority, and promulgation of direction to subordinates are events that couple the decision process to the rest of the activity of the force. The following probability statements are examples of the type of C3I requirements:

$\text{Pr}(\text{Data Obtained} \mid \text{communications connectivity, status of resources})$,
 $\text{Pr}(\text{Accurate Picture} \mid \text{Data Obtained, Accuracy of data, time delay, expectations})$,
 $\text{Pr}(\text{Recognize Situation} \mid \text{Accuracy of Picture, expectations})$,
 $\text{Pr}(\text{COA, resource, direction} \mid \text{Plan in place, situation as recognized, authority to act})$, and
 $\text{Pr}(\text{Decision promulgated} \mid \text{COA, communications connectivity, } \bullet)$.

These are conditioned on the plan that is in effect and how well it anticipated situations (expectations) and provided guidance for dealing with them (contingency plans). This suggests some requirements for planning:

$\text{Pr}(\text{Plan selected, in place} \mid \text{lead time})$, and
 $\text{Pr}(\text{Need to Replan} \mid \text{Plan in place, situation as recognized, Intelligence, etc.})$.

The time to accomplish these functions is inherently included in these requirements, often in terms of "timeliness". Component times may involve:

Time to obtain data,
Time to generate picture,
Time to recognize situation or need to replan,
Time to decide on course of action and resource allocation,
Time to promulgate decision, and
Time to replan.

With these types of C3I measures in place, along with Mission-oriented requirements measures, the overall outcome can be assessed with a view into the contribution of C3I, since the activation of Warfare Mission Area functions is conditioned on the direction to carry them out. The time to carry them out will not begin until they are initiated. Therefore, the achievement of the objectives depends on the probability of activation and the time to initiate action. This is explicitly the connection between C2 performance and the completion of Mission objectives.

SECTION 6.0 RELATIONSHIP TO OTHER SYSTEM ANALYSIS METHODS

This section discusses how the method described in this and the previous report is consistent or more general than some other approaches to system and decision analysis. Among these are Multi-Attribute Utility Theory (MAUT), Operational Sequence Diagrams and Petri nets. These methods are only mentioned here to introduce their role; more integration with these techniques will be discussed in a future report.

6.1 MULTI-ATTRIBUTE UTILITY THEORY

Utility Theory applies a measure of "worth" to each value of a single attribute. For each choice of alternative course of action, there is a probability distribution believed to hold with regard to which value of the attribute will be achieved. The expected "utility" of choosing that alternative is the sum of the products of the "worth" of an outcome value times the probability of that outcome. The alternative with the highest expected utility should be the "best" choice. But outcomes do not always involve single attributes, so Multi-Attribute Utility Theory weights the expected utility for each attribute to obtain an overall Utility. The weights are meant to convey relative importance of each attribute to the person making the choice. This pure form of MAUT uses marginal distributions of the probability of each attribute and implicitly assumes that the worth factors and importance weights are independent among the attributes. This is not always true, so there are variants of MAUT that heuristically address these situations.

A MAUT with full generality would assign worth to each joint outcome, conditioned on the chosen alternative, multiply these by the joint conditional probability distribution and sum over the joint outcomes to obtain expected utility conditioned on the choice. The Conditional Probability Logic provides the foundation for this approach with the utility analysis applied at the end. Another approach would use the assignment of worth, overlayed on the values of the joint outcomes, as a new utility outcome, which could be a new attribute whose probability would become the objective function.

Utility Theory is used to describe the motivation for decision making. There are different kinds of decisions to make. For example, there are Operational decisions and Acquisition decisions. The nature of the decision will affect the worth of outcomes or the importance weights of attributes. The Operational decision is one of choice of action and choice among resources available. The Acquisition decision is a choice of which resources to develop. The costs to the Operational decision maker are lives and production costs of losses. The Acquisition decision maker may consider those costs, too, but the development cost also is weighed. These costs are additional variables in the analysis. Importance weights can be used to put all costs and worth in balance or they can be a way of inferring why a particular alternative has been chosen by a decision maker. One can also use these weights to drive a decision to any choice one wants.

A decision maker at a certain level in the Hierarchy of Objectives will have certain attributes that are important at that level; other attributes that are part of the analysis will be a matter of indifference. In particular, the decision maker should be indifferent to the lower level outcomes that are aggregated to realize the outcomes at the higher level. But, to assess a decision at the higher level, it is necessary to have a means to infer the top level outcomes from the capabilities at the lower level. Aggregating probability over a causality net would provide a distribution of the top level outcomes, to which the worth of those outcomes could be applied. Lacking an ability to do that, current approaches use relative importance weights applied to a tree decomposition of outcomes rather than a causality net, and the capability values of low level functions are aggregated through importance weights rather than conditional probabilities. But the techniques and technology to take advantage of the causality approach are not mature enough to use. Petri nets, described in section

6.3, are a step in the direction of effective causality net analysis tools. Utility measures can be applied to the results of such an analysis tool.

6.2 OPERATIONAL SEQUENCE DIAGRAMS

Operational sequence diagrams (OSDs) are pictures of functional dependence among several resources in a system. Events at one point in the diagram may trigger processes in another. Time is shown explicitly as one dimension of the diagram, though not necessarily at a uniform scale. Resources are assigned portions of the other dimension. Symbols that indicate the beginning and end of processes within a resource are connected by lines in the time direction and lines connecting ends of a process to beginnings of processes in the same or another resource indicate dependence of the latter process on the product of the former.

OSDs are threads that weave through a causality net (see section 2.0), but they tend to treat individual outcomes rather than the whole range of values of the states simultaneously. If the range of outcomes at each branch point of an OSD is made mutually exclusive and exhaustive and the outcomes at other points or resources in the sequence depends on the full range of the outcomes at the antecedent branch, the OSD can be related to a causality net.

6.3 PETRI NETS

Petri nets are graphs of symbols representing events (transition symbols) and symbols representing states (place symbols) connected by directed arcs from transitions to places or places to transitions. Units, called tokens, move through the net, resting in places and being absorbed by transitions downstream, resulting in new ones being created there for placement in places downstream from the transition. These tokens can have attribute values associated with the transition which created them, in which case, the Petri net is called colored. If the probability of events is part of the consideration of whether a token is transitioned, it is a stochastic Petri net. If time is used as a determinant of events, it is a timed Petri net.

The causality net is a generalization of a Petri net, particularly if the Petri net is stochastic, attributed (or colored) and timed. Computer programs that implement Petri nets are emerging as analysis tools that will lead to capabilities to enable more complex system assessments.

SECTION 7.0 CONCLUSIONS

Hierarchy of Objectives and the approach of defining objects, attributes, processes and probabilities provide a complete, consistent and uniform method of top-to-bottom analysis, qualitative system description and quantitative bottom-to-top assessment. Conditional probabilities and time provide the only common measures for aggregating performance. But the tools to accomplish such an assessment are still under development. In addition, the complexity of the problem exceeds that of hydrodynamic modeling, so there is a long way to go.

In the meantime, it is important to understand the role of decision making in the chain of events that lead to Mission outcome. In many analyses, the conduct of operations is assumed to be activated because that is what the scenario is about. Sequences of events are postulated as occurring in tandem because that is the way the analyst would do it. Except for man-in-the-loop simulators, most decision making is pre-scripted. Decision events must be treated in the same way that physical events are, i.e., ones that have a certain likelihood of occurrence. The activation of an activity should be conditioned on the decision to do so. Conversely, the decision to do so does not ensure that the activity happens and there may be a delay between the time of the decision and the time for the change in mode of the system to occur or be completed.

The likelihood of a particular decision outcome, in turn, depends on the outcomes of the subfunctions of the decision process, described in the Command Process Model (reference (b)). These are dependent on the information available, including knowledge of plans and the authority to carry out them out. C3I requirements need to be stated in terms of these outcomes and dependencies, which include the context of the requirement statement. That context must include the Mission objective. What is a right decision for one objective, may be a wrong decision for another. This is one of the reasons that it is difficult to specify C3I requirements in isolation.

SECTION 8.0 REFERENCES

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APPENDIX A

CONDITIONAL ALGEBRA AND LOGIC

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CONDITIONAL ALGEBRA AND LOGIC

1.0 INTRODUCTION

The following discussion of Conditional Algebra and Logic is excerpted from the paper, A Combat and Decision Model Based on Conditional Probability Logic (reference 18). It is provided for information about the more detailed mathematics underlying the methods described in the main report, but it is not necessary for continuity of ideas, since a summary is provided.

Note that the word, "object", as used in this appendix, means a mathematical construct, not the "object" described by states and functions in the main report. The term, "object" or "conditional object", in this appendix is the same as the "conditional form" discussed in the main body of the report. In identifying states in this appendix, the symbol, T, represents information states as defined in the previous report on the Hierarchy of Objectives (reference 19). The symbol, N, represents physical states. The symbol, S, is for stimuli, while R stands for emissions (external responses). States of the environment are denoted by E or G depending on whether they represent phenomena or background (e.g., geographic) reference points, respectively.

2.0 ALGEBRA OF CONDITIONAL OBJECTS

The approach relies on a new Algebra of Conditional Objects, devised by Dr. Irwin R. Goodman of the Naval Ocean Systems Center, San Diego, in collaboration with Dr. Hung T. Nguyen of New Mexico State University, Las Cruces (references 1 through 4). The algebra defines a space of conditional events, conditional objects, and associated logical operators. This algebra is an extension of ordinary set algebras. When Probability measures are used to impose a logic on this algebra, the result is a Conditional Probability Logic. Other conditional logics are possible.

The mathematical foundation defines Conditional Objects and associated operations (e.g., intersection, union, negation) in a complete algebra. This algebra provides the syntactic or qualitative portion of the technique. A great deal of system definition can be accomplished without reference to the quantitative aspect, because the algebra addresses measure-free conditioning. Because of this, the structure of the design, described in the following paragraphs, is independent of the measure to be employed in calculating the results. The current technique takes advantage of the chain rule for assuring the qualitative independence (sufficiency statements) of conjunctive conditional events. Operations with Conditional Objects having different antecedents make it possible to deal with smaller data items.

A Conditional Object is an abstraction symbolized by $(X | Y)$, read "X given Y" or "X is conditioned on Y", which represents a dependence of a set of things (events or conditions (states)), X, on a set of things, Y. The Ys are called antecedents and the Xs are the consequents. Each capital letter represents a random variable that can take on values in some range. The values are represented by lower case symbols, x and y, if X and Y are continuous variables, or by subscripted versions, x_j and y_j , if they are discrete.

Goodman and Nguyen have devised an Algebra of Conditional Objects that defines operations on Conditional Objects without having to specify any particular P-measure on the events until a quantitative result is required. (A P-measure assigns a value between 0 and 1 to an event or set of events. Probability is the most common P-measure.) Some useful operations on Conditional Objects are listed below.

- a. An unconditioned object is actually conditioned on the whole problem space, i.e., $(X) = (X | \mathcal{R})$ or $(X) = (X | I)$.
- b. Important identities are $(X | Y) = (X, Y | Y)$ and $(X | X) = 1$. (The comma means "and".)
- c. A joint object (X, Y) can be obtained without resorting to independence by $(X, Y) = (X | Y) * (Y)$ or $(Y | X) * (X)$, where $*$ is "multiply", "intersection" or "join". Furthermore, the joint conditional object $(X, Y | Z) = (X | Y, Z) * (Y | Z) = (Y | X, Z) * (X | Z)$.
- d. One of the variables of a joint consequent can be removed by "integrating" over the values of that variable, i.e., the union (or sum) over y of $(X, Y | Z)$ is $(X | Z)$.
- e. Conditional independence results in $(X, Y | Z) = (X | Z) * (Y | Z)$, when $(X | Z)$ and $(Y | Z)$ are conditionally independent.
- f. If the object $(X | Y)$ is independent of (Z) , then $(X | Y, Z) = (X | Y)$. This is called a "sufficiency condition" for modeling purposes. (See Defining Functions and Submodels, section 4.) It also means that knowledge of (Y) is sufficient to infer (X, Y) or (X) , and that (Z) adds no information.

3.0 CONDITIONAL PROBABILITY LOGIC

The selection of a P-measure imposes a semantic or quantitative logic structure. It augments the Algebra with additional theorems about quantitative operations. When classical probability and conditional algebra are chosen, the result is a Conditional Probability Logic. When the P-measure is applied, the objects and the P-measure function notation are shown as equivalent, i.e., $P((X | Y)) = P(X | Y) = (X | Y)$.

Other conditional logics can be devised by applying other measures, such as possibility (reference 5) (conditional fuzzy logic), to the conditional object set. Any system model defined on the basis of the algebra will have the same structure, whether CPL or other logic is chosen. Only the quantitative aspects will differ; that is, "multiply" and "add" will be executed in accordance with the operations for that measure and logic. Models based on possibility, belief (reference 6), and truth measures are alternative or complementary to the CPL approach. Models with mixed approaches will probably be necessary. Conditional Logics have significant consequences for Artificial Intelligence, System Analysis and Neural Network technologies. (They are the "right" logics to use for inference; "implication" being the "wrong" approach. (reference 7))

4.0 DEFINING FUNCTIONS AND SUBMODELS

In $(X | Y)$, the dependence of consequents, X , on the set of antecedents, Y , can be viewed as a function with input events, Y , and output events, X . Functions can be connected together in series and parallel to form larger functions. Conversely, a function may be a subfunction of another function. There are equivalent and analogous aggregation and decomposition operations with conditional objects and probability. When a larger function is decomposed into subfunctions, not all the inputs to the larger function are needed by every smaller function. Instead, a sufficiency condition exists for the smaller function. (If the sufficiency condition $(X | Y, Z) = (X | Y)$ is assumed, then the function with outputs, X , does not need the inputs, Z .) This suggests the equivalence between the sufficiency conditions and the nature of the subfunction and information structure. The sufficiency condition or its functional equivalent is also referred to as a submodel.

A representation of structure by means of incidence matrices and directed graphs has been devised to relate the mathematical formulation of the CPL Model and its design structure. (See section 5.0.)

In a model, inputs, outputs, and functions are all represented by arrays of probability distribution values. These arrays have as many dimensions as they have antecedents and consequents. Each dimension is indexed over the range of the variable represented by that antecedent or consequent. For a given combination of antecedent indices, the sum over all the consequent indices must be 1.0. The entire run time operation of a CPL Model consists of a series of multiplications and integrations of these arrays, much like matrix operations.

5.0 DIRECTED GRAPHS AND INCIDENCE MATRICES

Directed graphs and incidence matrices are used to provide insights into model and system structure (reference 8). A directed graph is a set of nodes (graph nodes, not model nodes, which are called "objects" in the main report) connected by arcs with arrows from one node to another. (See Figure A-1, Example of a Directed Graph.) The arrow points in the direction of a "relation" between the nodes. For CPL modeling, the nodes represent event sets and the arcs represent the relation, "is the antecedent of" or "causes". For a given node, all the incoming arrows are coming from the several antecedent event sets of that consequent event set(s). When the nodes represent a single consequent, the graph is a representation of the functional relationship of the system and the "information" structure inherent in the submodel dependencies. This directed graph of conditional object relations is the simplest representation of any simulation of those event sets and their causal interdependencies. This approach was motivated by several papers by Ellett and Ericson (references 9 to 11), Howard and Matheson (reference 16) and Shachter (reference 17).

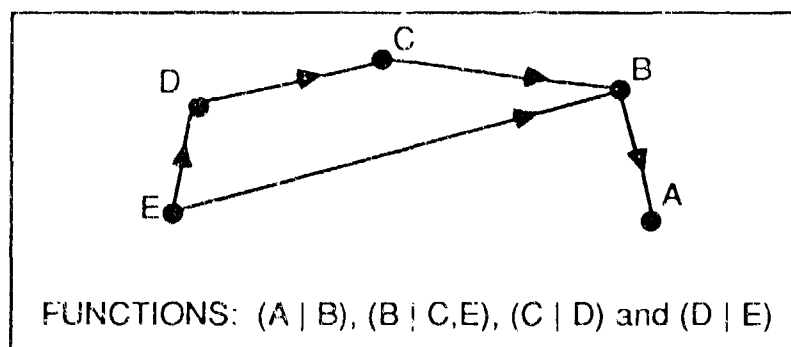


Figure A-1. Example of a Directed Graph

An Incidence Matrix is an array of 1s and 0s that indicates whether or not a node in a directed graph depends on another node. (Figure A-2, Example of an Incidence Matrix, shows the Incidence Matrix for Figure A-1.) These matrices can be used as the inputs to algorithms that analyze structure in terms of connectivity and higher order facets and characteristics of the graphs that represent the system being modeled (references 12 to 15). These algorithms could lead to methods for finding the degree of parallelism in the model that would aid in the mapping to parallel processors, such as neural nets. They may also help determine the most efficient sequence of combining the parts of the model in terms of memory requirements and processing time.

		ANTECEDENTS				
C O N S E Q U E N T S		A	B	C	D	E
	A	0	1	0	0	0
	B	0	0	1	0	1
	C	0	0	0	1	0
	D	0	0	0	0	1
	E	0	0	0	0	0

Figure A-2. Example of an Incidence Matrix

6.0 CPL MODELING APPROACH

A CPL Model calculates the probability distribution of events involving the internal processes of a set of Nodes (real world objects) and their external interactions through the Environment.

The Nodes have States, N, such as position, damage or number of missiles. These States are impacted by Stimuli, S, and are controlled by Decisions, T, which are also influenced by some of the Stimuli. The S, T and N sometimes cause emissions or Responses, R, which propagate through the Environment as "signals", E, which become Stimuli to the same or other Nodes in the setting. The "signals" may be energy fields, such as radar pulses, or material objects, such as missiles, which are not being modeled as nodes. Each S and E element is a representation of the same phenomenon. The Decisions involve choices in the desired state or responses, such as to change course or to fire a missile. All the S (or E), T, N and R may be viewed as "States" or Events of the entire setting, viewed as a "System".

A CPL Model implements a single step Markov transition model of the evolution of the distribution of these State events, but advantage is taken of the knowledge of structure and operations in the conditional algebra to obviate the effects of complexity, such as large memory and processing requirements.

6.1 DIRECT MARKOV MODELING

The direct Markov approach assumes that the initial distribution of states is known as well as the transition function for the evolution of the states. This transition function describes the conditional probability of moving from one combination of states to another combination in a certain time. (The prior set of events is called the antecedent set and the resulting set is the consequent set.) If the events are discrete the transition function and the initial distribution can be represented as multidimensional arrays of (conditional) probabilities. The process then involves multiplying the transition array and the initial distribution array and integrating over the set of initial combinations.

$$(N^*) = \int (N^* | N^-) \cdot (N^-) dN^- \quad (A-1)$$

In equation (A-1), the prior state distribution is represented by (N^-) , the current state distribution by (N^*) and the conditional transition distribution by $(N^* | N^-)$. Note that no $P(\dots)$ notation is used ahead of the variables in parentheses, (\dots) . The integral represents a multiple integral over all variables of integration, represented by dX , or in this case, dN^- .

Equation (A-1) represents the process of a closed system, that is, one with no inputs or outputs. It also does not show the role of decision making in the transition of states. In order to insert those factors, S , T and R events need to be modeled. This can be done by adding these "nuisance" variables to the process.

Adding S and T allows the calculation of the state evolution due to the effects of stimuli and decisions.

$$\begin{aligned} (N^* | N^-) &= \int (N^*, T^*, S^* | N^-) dT^* dS^* \\ &= \int (N^* | T^*, S^*, N^-) \cdot (T^* | S^*, N^-) \cdot (S^* | N^-) dT^* dS^* \end{aligned} \quad (A-2)$$

The factor, $(N^* | T^*, S^*, N^-)$, in equation (A-2), represents the aggregate model of the state transition functions as conditioned by stimulus, decision and prior state events. It is called a primitive function.^{1,2} The primitive function, $(T^* | S^*, N^-)$, represents aggregate or compound decision functions. The factor, $(S^* | N^-)$, is the input to the system and depends on the Environment conditions as discussed below. Primitive functions are listed in Table A-1.

TABLE A-1. PRIMITIVE FUNCTIONS

Responses	$(R^* N^*, T^*, S^*, N^-)$
Node States	$(N^* T^*, S^*, N^-)$
Decisions	$(T^* S^*, N^-)$
Environment	$(E^* G, R^*, N^*)$

The Environment is generated by the Responses of the Nodes and depends on their States. The Environment is represented by $(E | G)$, where E represents environment conditions (states) that are sensible (in the sensing sense), such as radar energy level, and G represents parametric variables of the background (G for Ground), such as location. (The conditional form is used because it behaves mathematically like a conditional probability, as will be seen.) In order to determine the Environment, the model calculates

$$(E^* | G) = \int (E^* | G, R^*, N^*) \cdot (R^*, N^*) dR^* dN^* \quad (A-3)$$

The primitive function, $(E^* | G, R^*, N^*)$, is the model of the Environment. But in order to calculate equation (A-3), the distribution, (R^*, N^*) is needed. This can be obtained by

$$\begin{aligned}
 (R^*, N^*) &= \int (R^*, N^*, T^*, S^* | N^-) \cdot (N^-) dT^* dS^* dN^- \\
 &= \int (R^* | N^*, T^*, S^*, N^-) \cdot (N^* | T^*, S^*, N^-) \\
 &\quad \cdot (T^* | S^*, N^-) \cdot (S^* | N^-) \cdot (N^-) dT^* dS^* dN^-
 \end{aligned}
 \tag{A-4}$$

The primitive function for Response is the first factor in the second part of equation (A-4).

The input to the process, $(S^* | N^-)$, can be obtained by equating it to the environmental condition where the background parameters, G , has the value of the State, N .

$$\begin{aligned}
 (S^* | N^-) &\stackrel{d}{=} (E^* | G) \\
 &\quad | G=N^- \\
 &= \int (E^* | N^-, R^*, N^*) \cdot (R^*, N^*) dR^* dN^*
 \end{aligned}
 \tag{A-5}$$

This closes the cycle back to equation (A-2), allowing the state transition array to be calculated. (The role of N^* and N^- are not clearly differentiated here. A great deal of care needs to be paid to the relative time used in the problem. Time is sometimes "in the interval, t_k ", and sometimes it means "at the beginning or end of the interval". This is addressed in each decomposition of the primitives separately.) The result of equation (A-5) can then be inserted in equation (1) to generate the next state distribution.

This approach is intractable, however, when the dimensionality and range of the variables is considered. An attack on the complexity problem will be discussed in the next section. The graphs of the components of equations (A-2) through (A-4) are shown in figures A-3 through A-5. Figure A-4 also shows the equivalence identified in equation (A-5).

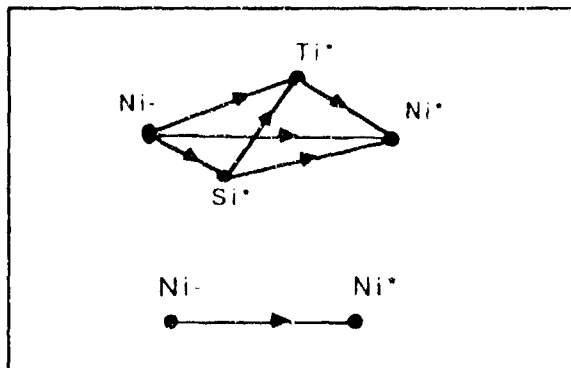


Figure A-3. Directed Graph for Equation (A-2)

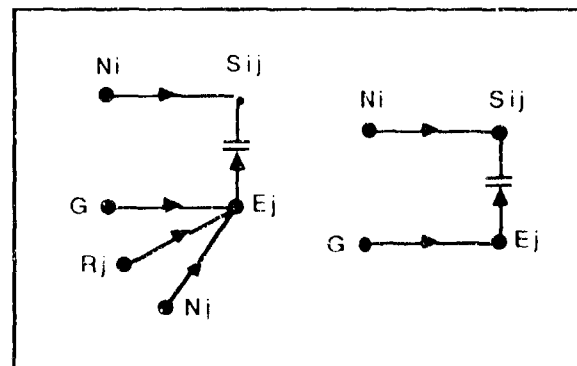


Figure A-4. Directed Graph for Equation (A-3)

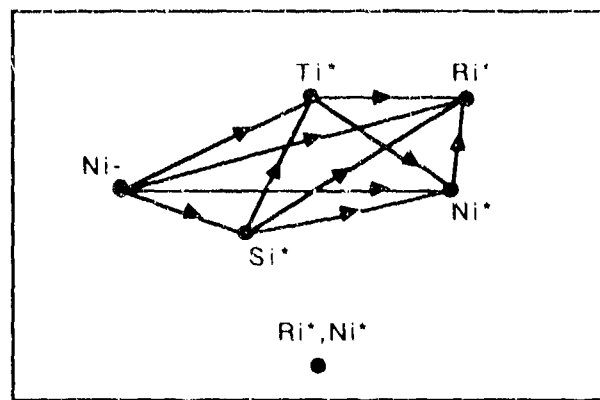


Figure A-5. Directed Graph for Equation (A-4)

6.2 SUFFICIENCY MODELING (OR STRUCTURAL MODELING)

The S, T, N and R events are made up of several event subtypes. The number of combinations of these grows geometrically (the product of the size of the ranges of each subtype). If all events depended directly on all others, the direct Markov approach would be intractable. The size of the initial distribution array is the product of the number of distinct events in each state subtype and the size of the transition array is the square of that. The primitive function arrays are even larger. But certain events (consequents) only depend directly on a subset of the others. The subset consists of the antecedents of the dependent events. Then knowledge of the distribution of the antecedent subset and the conditional distribution that relates the antecedents to the consequents is sufficient to determine the distribution of the consequents. This is called a "sufficiency axiom". This is a form of independence statement. When there is only one subtype in the consequent, the small transition array that relates the sufficient antecedents to that consequent is called a submodel array. This submodel represents a single "function" in the "system". The submodel also defines the direct causality among the events of the "system". Indirect causality is the result of the "chain of events" that results from the concatenation of the submodels.

The Algebra of Conditional Objects allows the multiplication of these submodel arrays together to form the aggregate primitive functions. (Since the antecedents of these arrays are different, there was no previous mathematical justification for this multiplication.) The primitives could then be used in equations (A-1) to (A-5) to find the new State distribution. However, this would again lead to the intractable procedure. Instead of building up to doing one massive multiplication and integrating over the entire combination of "states", as in equation (A-1), one can multiply the small arrays together in a judicious sequence, integrating over a small set of the dimensions of the problem along the way. This reduces array sizes and the number of operations significantly.

The first level of independence statement is that the nodes are only influenced by "signals" in the environment, not directly by the states of the other nodes and, therefore, the node state distributions are separately calculable. At the next level, the signals are assumed to be independent of each other (non-interfering) and, within each Node, certain of the S, T, N and R events are only directly dependent on a small set of the others. These intranode dependencies and the propagation functions are all defined by submodel arrays.

The design of a model involves defining the states (dimensions), the range of their values, and the functions (sufficiency axioms) of the system (the nodes and the environment), developing a program that fills in the values of conditional probability in the submodel arrays that define the

causal dependence of those functions and determining the most judicious sequence of combining and integrating those arrays in successive cycles. Within each cycle, certain joint state-response distributions are determined in order to generate the conditional distribution of the environmental states, which are then used to stimulate the nodes. The output of the model is a sequence of joint state distributions for each node for each cycle. These distributions are marginal ones relative to the multinode "system". Marginal distributions can be reduced to fewer dimensions by integrating over states that are not of concern to the analysis of objectives. This reduces insight into the result in those dimensions.

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APPENDIX B

EFFECT OF COUNTERTARGETING DECISIONS

APPENDIX B

EFFECT OF COUNTERTARGETING DECISIONS

The question addressed by this section is how decisions to countertarget the weapon systems of submarine-launched cruise missiles (SLCM) contribute to the Mission Success Criterion (MSC) of "limiting loss" of own force units. An objective factor in such an analysis is to minimize the number of missiles launched by the enemy as a proportion, p , of how many are carried. The complement of this number, $1-p$, is not a measure of our countertargeting effectiveness. The idea that our countertargeting effectiveness is the complement of the percent of weapons that the enemy launches is simplistic and misleading. It aggregates, inseparably, the enemy's targeting capability and our actual countertargeting effectiveness. In addition, it fails to differentiate between the enemy's capability when we are attempting to countertarget and that capability when we are not. The analysis that follows accounts for these things and it shows the role of the tactical decision in the process. The enemy's decision to attempt to target and launch will also be accounted for. It will also be noted that the simplistic approach is tantamount to assuming that the enemy's targeting capability is perfect and that we always attempt to countertarget the enemy, whether or not that is a wise move.

This analysis is based on simple binomial probabilities on the occurrence of events (A occurs or NOT A occurs). These probabilities are conditioned on the occurrence of prior events. Figure B-1 shows the kinds of events to be considered. Data obtained or systems activated are shown in rectangles, while decisions to be made are in diamonds. The top row of symbols represents own force events, while the lower part is the enemy's set. The context of the analysis is in the shaded area. The enemy has recognized an opportunity to attack (based on external surveillance) and is attempting to launch, but must first gain contact and classify the contact and achieve attack criteria. Our surveillance suggests that we are threatened and we have a choice to employ countermeasures to deceive the attacker into thinking that the contact is not a carrier. Deception effectiveness can then be reflected in the enemy's belief that it is not a carrier. The enemy also has to achieve attack criteria and the deception may increase or decrease the enemy's capability to localize. Intelligence information concerning the enemy's capabilities in the presence or absence of countermeasures is in the data base.

Suppose that the sets of events involved in the analysis is represented by the Venn diagram of figure B-2. Table B-1.1 provides a list of symbols that will be used in this analysis and their meanings. The set, $\{LA\}$, represents launch attempts; that is, the analysis starts with the condition that the enemy is attempting to launch. The inner circle is the set, $\{LAS\}$, of successful launch attempts. The left side (set $\{CA, LA\}$) of the set, $\{LA\}$, represents those launch attempts that are opposed by our countering effort. This includes the decision to countertarget and the activation of the countermeasures. The top half (set $\{CAS, CA, LA\}$) of the left side represents successful countertargeting, when it is attempted, while the bottom half (set $\{CAF, CA, LA\}$) represents unsuccessful countertargeting attempts. (Notice that, even when we do not attempt to counter, the enemy is not always successful at achieving launch. Furthermore, although counters may have been unsuccessful when attempted, the enemy still may not have achieved launch in 100 percent of set $\{CAF, CA, LA\}$. However, when countertargeting is attempted and is successful, the enemy achieves no launches, since the deception would cause a decision not to order the launch.)

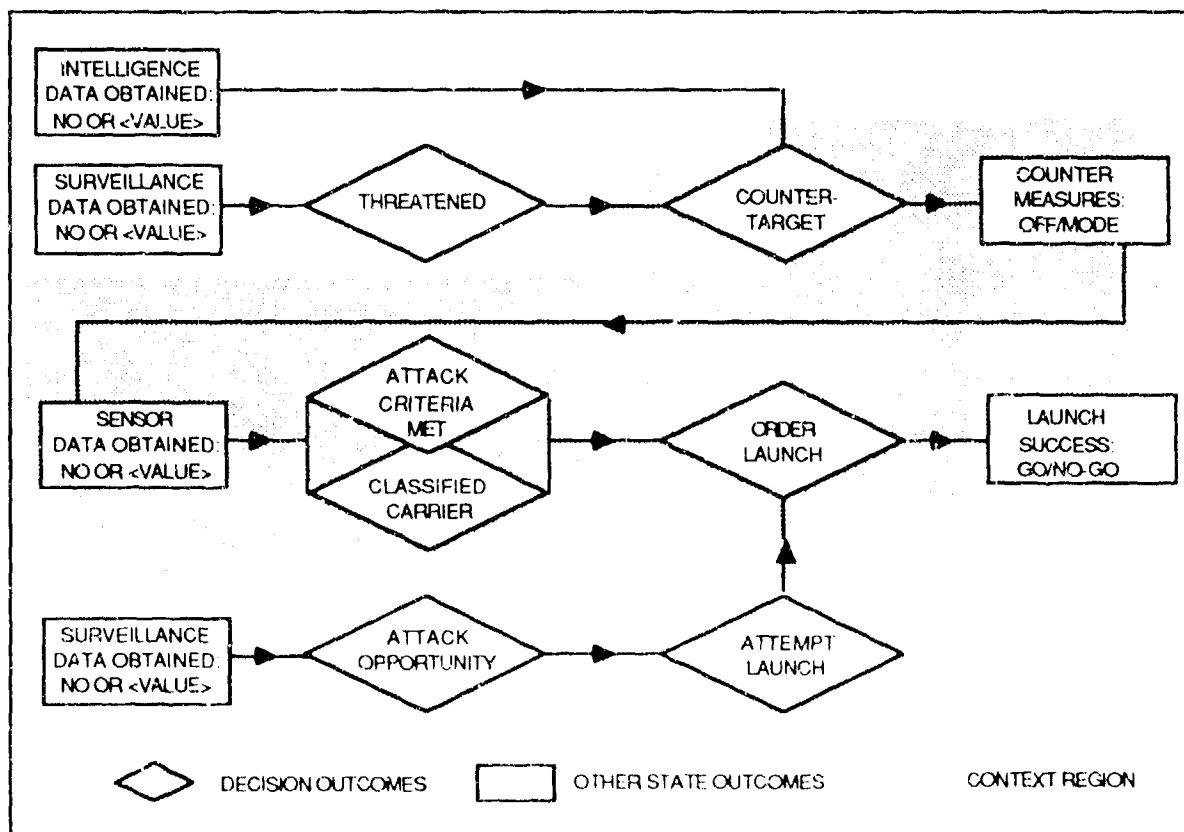


Figure B-1. Relationship of States in Countertargeting Example

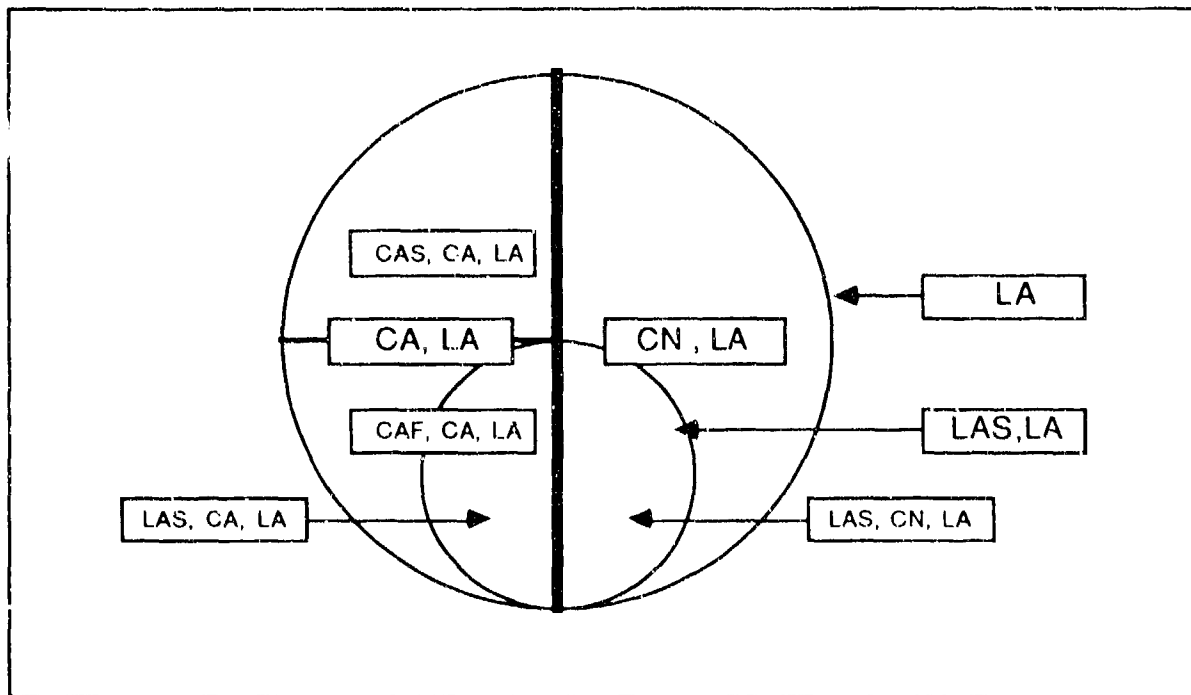


Figure B-2. Venn Diagram for Launch Attempt

TABLE B-1.1 SYMBOLS AND MEANINGS

<u>SYMBOL</u>	<u>MEANING</u>
LA	Launch is attempted by the enemy
LN	Launch not attempted " " "
LAS	Launch attempt is successful
LAF	Launch attempt failed
CA	Countertargeting is attempted (by us)
CN	Countertargeting is not attempted
CAS	Countertargeting attempt is successful
CAF	Countertargeting attempt fails

The relationship among these sets can be represented by conditional probabilities. The overall probability of launch by the enemy, given that he is attempting to launch, can be written as $\Pr(\text{LAS} | \text{LA})$. This is the objective factor for this context. For this analysis, let it be called the launch ratio, LR, since it is the ratio of the area of the set {LAS} to that of the set {LA}.

A number of other key factors can be represented by conditional probabilities. Table B-1.2 lists these factors, by symbol, by descriptor, and by conditional probability expression. These expressions are read as:

$\Pr(A,B | C,D)$ means Probability of A AND B, given C AND D.

TABLE B-1.2. ANALYSIS FACTORS

SYMBOL	DESCRIPTOR	PROBABILITY
LR	Launch Ratio (Overall)	$\text{LAS} \text{LA}$
OLR	Opposed Launch Ratio	$\text{LAS} \text{CA}, \text{LA}$
ULR	Unopposed Launch Ratio	$\text{LAS} \text{CN}, \text{LA}$
CTAR	Countertargeting Attempt Ratio	$\text{CA} \text{LA}$
CTSR	Countertargeting Success Ratio	$\text{CAS} \text{CA}, \text{LA}$
ROLR	Residual Opposed Launch Ratio	$\text{LAS} \text{CAF}, \text{CA}, \text{LA}$

The probabilities listed in Table B-1.2 are conditional probabilities. These can be calculated from the areas of a Venn diagram (not to scale) by the equation:

$$\Pr(A | B) = \Pr(A,B) / \Pr(B) \quad (1)$$

Therefore, the Launch Ratio becomes

$$\text{LR} = \Pr(\text{LAS} | \text{LA}) = \Pr(\text{LAS}, \text{LA}) / \Pr(\text{LA}) \quad (2)$$

which is the ratio of the area of the smaller circle to that of the larger one.

The region, {LAS,LA}, is made up of the subareas, {LAS,CA,LA} and {LAS,CN,LA}. Because of this

$$\Pr(\text{LAS}, \text{LA}) = \Pr(\text{LAS}, \text{CA}, \text{LA}) + \Pr(\text{LAS}, \text{CN}, \text{LA}) \quad (3)$$

The calculation of the enemy's contributing launch ratios results from

$$\text{OLR} = \Pr(\text{LAS} | \text{CA}, \text{LA}) = \Pr(\text{LAS}, \text{CA}, \text{LA}) / \Pr(\text{CA}, \text{LA}) \quad (4)$$

when opposed by our CT attempts, and

$$\text{ULR} = \Pr(\text{LAS} | \text{CN}, \text{LA}) = \Pr(\text{LAS}, \text{CN}, \text{LA}) / \Pr(\text{CN}, \text{LA}) \quad (5)$$

when not opposed.

The rate of success of our countertargeting attempts; that is, when we actually cause a failure in the enemy's targeting and launch sequence, is given by

$$\text{CTSR} = \Pr(\text{CAS} \mid \text{CA}, \text{LA}) = \Pr(\text{CAS}, \text{CA}, \text{LA}) / \Pr(\text{CA}, \text{LA}) \quad (6)$$

The final ratio of interest is the proportion of time the enemy is able to launch when opposed by our countertargeting attempts, but we were unsuccessful in causing any failure. The enemy may not always achieve launch in this case, due to other failures in the enemy system not caused by us. This Residual Opposed Launch Ratio is

$$\begin{aligned} \text{ROLR} &= \Pr(\text{LAS} \mid \text{CAF}, \text{CA}, \text{LA}) \\ &= \Pr(\text{LAS}, \text{CAF}, \text{CA}, \text{LA}) / \Pr(\text{CAF}, \text{CA}, \text{LA}) \\ &= \Pr(\text{LAS}, \text{CA}, \text{LA}) / \Pr(\text{CAF}, \text{CA}, \text{LA}) \end{aligned} \quad (7)$$

since the set $\{\text{LAS}, \text{CAF}, \text{CA}, \text{LA}\}$ is, in this case, the same as the set $\{\text{LAS}, \text{CA}, \text{LA}\}$. This is because what would have been set $\{\text{LAS}, \text{CAS}, \text{CA}, \text{LA}\}$ is empty since launch cannot be successful when CT is totally successful. (Less than total success is considered a failure. Partial success contributes to ROLR but requires a decomposition of the CT process, which is beyond the scope of this analysis.)

ROLR can be written in terms of OLR and CTSR by continuing equation (7):

$$\begin{aligned} \text{ROLR} &= \Pr(\text{LAS} \mid \text{CA}, \text{LA}) * \Pr(\text{CA}, \text{LA}) / \Pr(\text{CAF} \mid \text{CA}, \text{LA}) * \Pr(\text{CA}, \text{LA}) \\ &= \Pr(\text{LAS} \mid \text{CA}, \text{LA}) / [1 - \Pr(\text{CAS} \mid \text{CA}, \text{LA})] \\ &= \text{OLR} / [1 - \text{CTSR}] \end{aligned} \quad (8)$$

Now, consider another way of calculating the launch ratio, LR. In this case,

$$\begin{aligned} \text{LR} &= \Pr(\text{LAS} \mid \text{LA}) = \Pr(\text{LAS}, \text{LA}) / \Pr(\text{LA}) \\ &= [\Pr(\text{LAS}, \text{CA}, \text{LA}) + \Pr(\text{LAS}, \text{CN}, \text{LA})] / \Pr(\text{LA}) \\ &= [\Pr(\text{LAS} \mid \text{CA}, \text{LA}) * \Pr(\text{CA}, \text{LA}) \\ &\quad + \Pr(\text{LAS} \mid \text{CN}, \text{LA}) * \Pr(\text{CN}, \text{LA})] / \Pr(\text{LA}) \\ &= [\Pr(\text{LAS} \mid \text{CA}, \text{LA}) * \Pr(\text{CA} \mid \text{LA}) * \Pr(\text{LA}) \\ &\quad + \Pr(\text{LAS} \mid \text{CN}, \text{LA}) * \Pr(\text{CN} \mid \text{LA}) * \Pr(\text{LA})] / \Pr(\text{LA}) \\ &= \Pr(\text{LAS} \mid \text{CA}, \text{LA}) * \Pr(\text{CA} \mid \text{LA}) \\ &\quad + \Pr(\text{LAS} \mid \text{CN}, \text{LA}) * \Pr(\text{CN} \mid \text{LA}) \end{aligned} \quad (9)$$

Replacing the probabilities with their symbols from Table B-1.2 yields:

$$\begin{aligned} \text{LR} &= \text{OLR} * \text{CTAR} + \text{ULR} * [1 - \text{CTAR}] \\ &= \text{ROLR} * [1 - \text{CTSR}] * \text{CTAR} + \text{ULR} * [1 - \text{CTAR}] \end{aligned} \quad (10)$$

Equation (10) is a fully explicit representation of the relationship between our countertargeting capabilities, CTSR, the enemy's targeting capabilities, ULR and ROLR (or GLR), and our decision to conduct countertargeting operations, CTAR. (The enemy's decision to attempt launch will be considered later. For now, this analysis is focused on just the set of launch attempts.)

Let's look at some simplified versions of equation (10) by making some simplifying assumptions.

$$\begin{aligned} \text{Let ROLR} &= \text{ULR} && \text{(Assumption A)} \\ \text{Let CTAR} &= 1.0 && \text{(Assumption B)} \end{aligned}$$

Assumption (A) makes the enemy's launch capability the same whether he is unopposed or we fail in our countering attempt. Under this assumption, equation (10) becomes:

$$\begin{aligned} LR &= ULR * [(1 - CTSR) * CTAR + (1 - CIAR)] \\ &= ULR * [1 - CTAR * CTSR] \end{aligned} \quad (11)$$

Assumption (B) says that we always attempt to counter when the enemy attempts to launch. Under these assumptions:

$$LR = ULR * [1 - CTSR] \quad (12a)$$

This is the simplified two-sided game, where:

$$\begin{aligned} \{ \text{Probability of Launch} \} \\ &= \{ \text{His Launch Capability} \} \\ &\quad * \{ 1 - \{ \text{Our Countertargeting Capability} \} \} \end{aligned}$$

Equation (12a) can be rewritten as:

$$CTSR = 1 - [LR / ULR] \quad (12b)$$

(Note that equation (12) fails if $LR > ULR$. This is because our assumption that $ROLR = ULR$ was false in that our attempt to counter his launch actually helped him and $ROLR > ULR$. We will deal with this later.)

Now if we make the extreme assumption that his launch capability is perfect when unopposed or if we fail to attempt countertargeting:

$$ULR = ROLR = 1.0 \quad (\text{Assumption C})$$

Then the result is the perception that the countertargeting effectiveness is the complement of the launch ratio:

$$CTSR = 1 - LR = 1 - p \quad (13)$$

when $LR = p$.

This lends little insight into the role of our capabilities and assumes all the burden is on our forces. Equation (12) shows that the enemy's capabilities can be balanced against ours and equation (10) shows how we can combine the results for when we are attempting to counter and when we are not. Table B-1.3 shows some examples of balancing equation (12). In the fourth line, there is an example where postulating a perfect enemy launch capability, $ULR = 1.0$, and a CTSR of 0.33 yields a percent-launched of 0.67. That same CTSR in the ninth line, along with a reasonable ULR of 0.75, yields a 0.50 launch ratio. Conversely, a 0.67 launch rate can also result for an enemy ULR of 0.75 (line 6), which only requires a 0.22 CTSR. This is less than that required in equation (13), which is the equivalent of the fourth line of Table B-1.3.

TABLE B-1.3. BALANCING EQUATION (12)

<u>LR</u>	<u>ULR</u>	<u>CTSR</u>
0.750	1.000	0.250
0.750	0.900	0.167
0.750	0.750	0.000
0.667	1.000	0.333
0.667	0.900	0.256
0.667	0.750	0.222
0.500	1.000	0.500
0.500	0.900	0.445
0.500	0.750	0.333

Note: CTAR = 1 and ROLR = ULR

But equation (12) assumed that we always attempted to counter, i. e., CTAR = 1. This may not always be the case, particularly if we do not know he is there, an event highly dependent on our cueing and/or our organic detection capabilities. It may also be a decision based on our expectation of the effect of our countering efforts on the enemy targeting capability, especially if those efforts might enhance that capability, as we shall see later. For now, let's relax assumption (B) and rewrite equation (11).

$$CTSR * CTAR = 1 - [LR / ULR] \quad (14)$$

(Note that equation (14) can fail for the same reason that equation (12) might.)

Table B-1.4A shows the balancing of equation (14) with the enemy's launch capability, ULR = ROLR, held constant at 0.75 in order to compare with the examples of Table B-1.3, lines 6 and 9. Notice that the required CTSR is higher now since CTAR has been reduced (< 1). In particular, when CTAR = 0.67, CTSR again needs to be 0.33.

TABLE B-1.4A. BALANCING EQUATION (14)

<u>LR</u>	<u>CTAR</u>	<u>CTSR</u>
0.750	0.900	0.000
0.750	0.750	0.000
0.750	0.667	0.000
0.667	0.900	0.247
0.667	0.750	0.296
0.667	0.667	0.333
0.500	0.900	0.370
0.500	0.750	0.444
0.500	0.667	0.500

Notice that equation (14) isolates, to the left side of the equation, both of the variables that are under our control, CTSR and CTAR. The total result, LR, and the enemy's capability, ULR, are on the right. Now we can do some trade-offs. Increasing CTSR for a given LR and ULR allows a decrease in CTAR and vice versa. If better cueing or detection increases CTAR, then less CTSR capability is needed.

Tables B-1.4B and 1.4C show an extreme set of values for CTAR and CTSR. If we could achieve a perfect CTSR of 1.0, our minimum requirement for attempting countertargeting is a function of the goal value of LR which we are trying to stay under. If we always attempt to counter (CTAR = 1.0), these same values arise as the minimum countertargeting effectiveness. (Note that, when the enemy's capability, ULR, and our goal, MAX LR, are the same, no countering attempt or capability is required.)

TABLE B-1.4B. EXTREME VALUES FOR PERFECT CTSR

MAX LR	MIN CTAR	PERFECT CTSR
0.750	0.000	1.000
0.667	0.222	1.000
0.500	0.333	1.000

TABLE B-1.4C. EXTREME VALUES FOR ALWAYS ATTEMPTING CT

MAX LR	ALWAYS CTAR	MIN CTSR
0.750	1.000	0.000
0.667	1.000	0.222
0.500	1.000	0.333

Note: ULR = 0.750 for all of TABLES B-1.4A, B, C.

Now, let's examine what happens when we relax assumption (A). Returning to equation (10):

$$\begin{aligned} LR &= OLR*CTAR + ULR*[1 - CTSR] \\ &= ROLR*[1 - CTSR]*CTAR + ULR*[1 - CTSR] \end{aligned} \quad (10)$$

First, let's look at the product of ROLR and [1 - CTSR]. Some would say, if [1 - CTSR] is the probability that we failed to countertarget, then should ROLR be 1.0? Or, what is the meaning of [1 - CTSR] if ROLR is not 1.0? CTSR is our capability to cause failure when the enemy would otherwise be successful; ROLR is only the probability that he would be successful when we fail. When we fail to cause his failure, this probability is reduced by the same internal deficiencies that could cause failure to launch, as reflected in ULR, independent of our attempts and failure to countertarget.

On the other hand, our attempts to counter could even enhance ROLR over ULR, by making attack criteria easier to achieve due to the presence of the deception signal. ROLR may also be less than ULR if enough confusion was caused by the deception that it caused uncertainty in the enemy's decision. Since it keeps all the factors explicit, equation (10) is preferred over equations (12) and

(14) for doing trade-offs. In this case, understanding of the trade-offs can be viewed by rewriting equation (10) in simpler symbols. Let

$$\begin{aligned} z &= LR; \\ a &= ULR; \\ b &= ROLR; \\ x &= CTAR; \text{ and} \\ y &= CTSR. \end{aligned} \tag{15}$$

Then, equation (10) becomes

$$z = a + [b - a]x - bxy \tag{16}$$

Letting z^* be a threshold value for LR with a^* and b^* as values of enemy capabilities, then the set of values of x and y that just achieve the goal (threshold) lie under the hyperbolic curve $z = z^*$. Any values of x and y , that make $z \leq z^*$, are potential goals for our likelihood of initiating countertargeting and our capability once initiated.

Figures B-3 through B-5 are graphs of equation (16) for various values of a^* , b^* , and z^* . The shaded portion of each figure represents equation (16), while the horizontal surface, $z_2 = z^*$, cuts through the shaded surface on the skewed hyperbolic curve, $z_1 = z_2$. The goal values of x and y lie on the curve,

$$y = [(b^* - a^*)x - (z^* - a^*)] / b^*x, \tag{17}$$

which is a vertical projection onto the x - y plane of the intersection, $z_1 = z_2$. The reason that the three figures exhibit such different characteristics is that the relative magnitude of the three governing variables, a^* , b^* , and z^* , can occur in any order of relative magnitude. There are six such orders for inequality between all three variables, six orders for equality between two of the variables, and one for equality among all three. Whenever z^* is equal to or greater than both a^* and b^* , any values of x and y are acceptable. Figure B-3 represents the least eccentric case, where the enemy's ULR, a^* , exceeds his ROLR, b^* , which also exceeds the LR limits, z^* , that we hope to achieve with our countering efforts. Since ROLR is less than ULR, our attempts do not enhance his targeting. Since the LR goal, z^* , is less than both, we need countering capability to achieve our goal. To succeed, we need $x \geq x_1$ and $y \geq y(x)$. This is the region in the far right corner of the x - y plane. This suggests that the more likely we are to attempt to countertarget, the less capable our effort needs to be. This highlights a trade-off between alertment and countermeasures.

Figure B-4 shows the case when the confusion caused by our counter reduces the ROLR below the LR goal. Then the deception does not have to work completely. Use of countermeasures more often than x_2 requires no complete deceptions, since z_1 is always less than the goal.

Figure B-5, on the other hand, reflects the condition where ROLR is better than ULR. In this situation, if our deception does not work well, i.e., y is small, then use of countering is to be avoided, i.e., stay out of the lower right corner.

If Intelligence tells us that the situation of figure B-4 applies, i.e., ROLR low, but if figure B-5 is the actual case, i.e., ROLR high, the likelihood of a decision to use countermeasures would be high and the result would be disastrous, particularly since the enemy's ULR is below our goal in the latter case. This reflects the necessity for good Intelligence data base information, not only concerning the enemy's capability, but also about our effectiveness against it. If we think our deception is likely to succeed ($CTSR = 1$), it is always the case that we should use it, since the upper right corner of each figure shows $z_1 = 0$. This is because it is assumed that the enemy will always decide not to shoot if it is believed that the target is not a carrier.

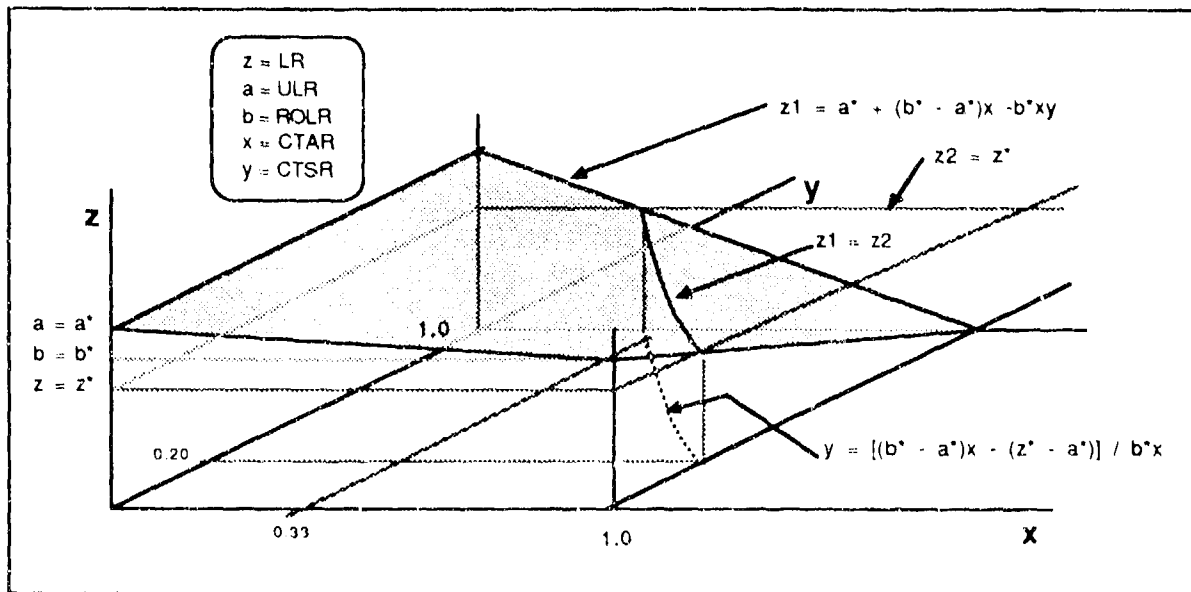


Figure B-3. Graph of z for $z^* < b^* < a^*$

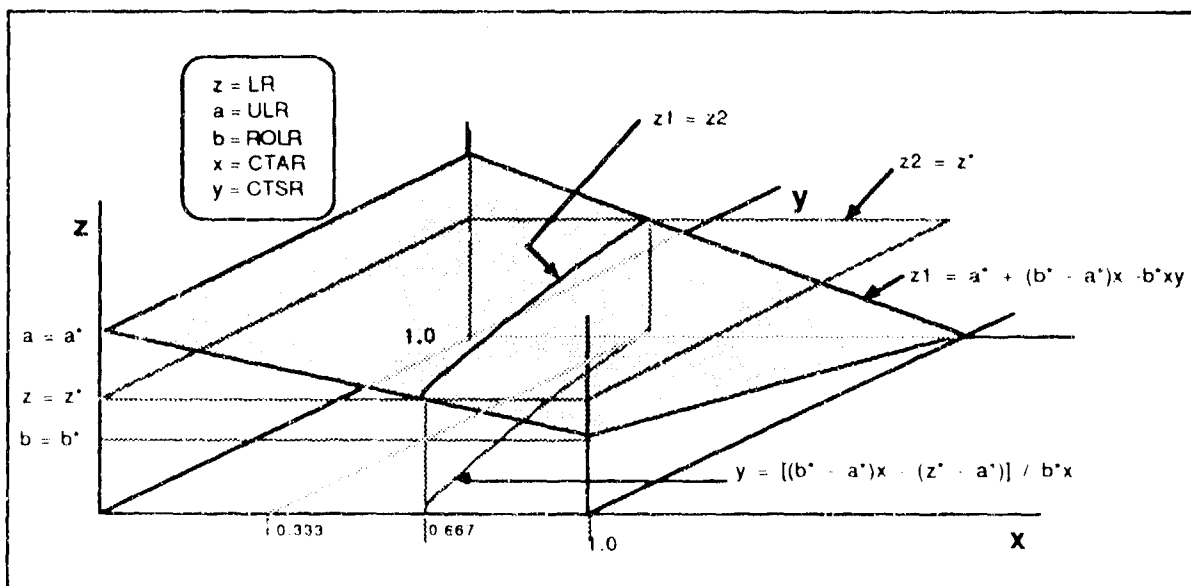


Figure B-4. Graph of z for $b^* < z^* < a^*$

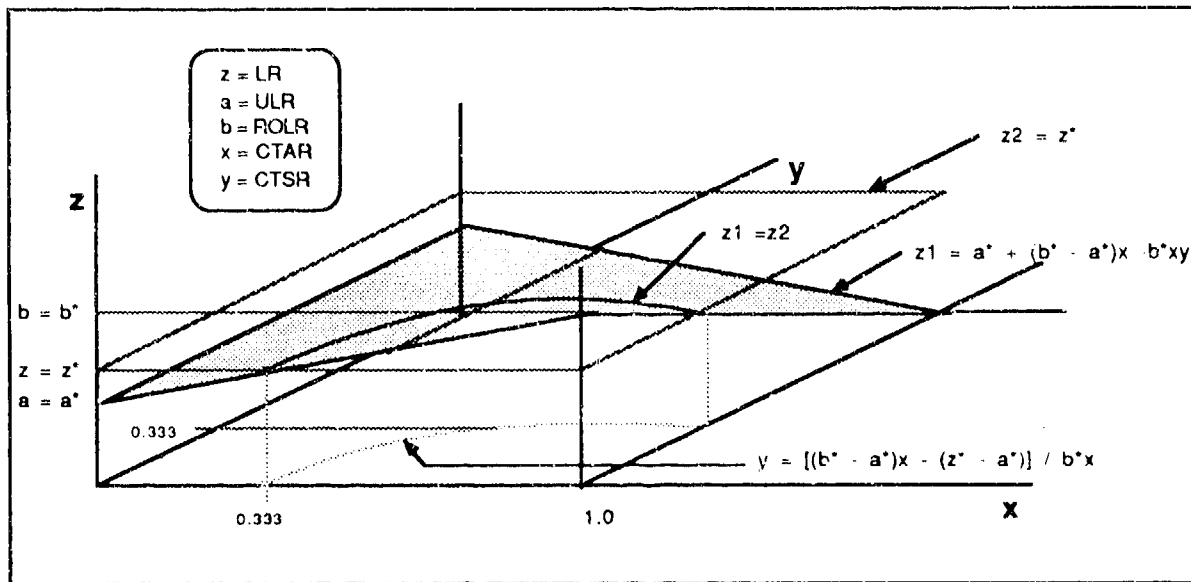


Figure B-5. Graph of z for $a^* < z^* < b^*$

It must be remembered that all of the above analysis was performed under the assumption that the enemy had already decided to attempt to launch. This assumption suppressed external aspects of the analysis, such as whether the enemy had an opportunity or the opportunity was known to the enemy. In other words, these were assumed for the purpose of focusing on the context as stated. The values above would have to be scaled by the probability that an opportunity existed and the enemy had decided to attempt a launch.

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